

Math 111: Summary of Trigonometric Identities

Reciprocal Identities

$$\begin{array}{lll} \sin \theta = \frac{1}{\csc \theta} & \cos \theta = \frac{1}{\sec \theta} & \tan \theta = \frac{1}{\cot \theta} \\ \csc \theta = \frac{1}{\sin \theta} & \sec \theta = \frac{1}{\cos \theta} & \cot \theta = \frac{1}{\tan \theta} \end{array}$$

Quotient Identities Pythagorean Identities Even/Odd Identities

$$\begin{array}{lll} \tan \theta = \frac{\sin \theta}{\cos \theta} & 1 = \sin^2 \theta + \cos^2 \theta & \sin(-\theta) = -\sin \theta & \csc(-\theta) = -\csc \theta \\ \cot \theta = \frac{\cos \theta}{\sin \theta} & \sec^2 \theta = \tan^2 \theta + 1 & \cos(-\theta) = \cos \theta & \sec(-\theta) = \sec \theta \\ & \csc^2 \theta = 1 + \cot^2 \theta & \tan(-\theta) = -\tan \theta & \cot(-\theta) = -\cot \theta \end{array}$$

Cofunction Identities

$$\begin{array}{lll} \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta & \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta & \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \\ \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta & \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta & \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta \end{array}$$

Sum and Difference Formulas

$$\begin{array}{ll} \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha & \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha & \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta & \\ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta & \end{array}$$

Double Angle Identities Power Reducing Identities Half Angle Identities

$$\begin{array}{lll} \sin 2\theta = 2 \sin \theta \cos \theta & \sin^2 \theta = \frac{1 - \cos 2\theta}{2} & \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} & \cos^2 \theta = \frac{1 + \cos 2\theta}{2} & \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \cos 2\theta = \cos^2 \theta - \sin^2 \theta & \tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta} & \tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta} \\ = 2 \cos^2 \theta - 1 & & \\ = 1 - 2 \sin^2 \theta & & \end{array}$$

Product to Sum Identities

$$\begin{array}{l} \sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\ \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)] \end{array}$$

Sum to Product Identities

$$\begin{array}{l} \sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right) \\ \sin \alpha - \sin \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right) \end{array}$$