L'Hôpital's Rule and Indeterminant Forms

Suppose f and g are differentiable and $g'(x) \neq 0$ near a (except possibly at a). Further, suppose that:

$$\lim_{x\to a} f(x) = 0 \quad \textit{and} \quad \lim_{x\to a} g(x) = 0$$
$$\textit{or}$$
$$\lim_{x\to a} f(x) = \pm\infty \quad \textit{and} \quad \lim_{x\to a} g(x) = \pm\infty.$$

Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Think of L'Hôpital's Rule as a hospital to fix these limits!

Indeterminant Forms

Emergency!	**Is there a doctor in the house?**	**Rush to the Hospital!**
$\left(\frac{\mathbf{0}}{\mathbf{0}}\right)$	Form satisfies L'Hôpital's Rule.	Apply L'Hôpital's Rule until you arrive at a **determinant** form.
$\left(\frac{\infty}{\infty}\right)$	Form satisfies L'Hôpital's Rule.	Same prescription as above.
$(\infty - \infty)$	Use algebra to rewrite the difference as a quotient by either finding a common denominator or multiplying by the conjugate.	Same prescription as above.
$(\mathbf{0} \cdot \infty)$	Use algebra to rewrite the product as a quotient $$f(x)g(x) = \frac{f(x)}{1/g(x)}$$	Same prescription as above.
$(\mathbf{0}^{\mathbf{0}})$ $(\infty^{\mathbf{0}})$ $(\mathbf{1}^{\infty})$	Use rules of logarithms to rewrite the power as a product or a quotient. For example, $$\lim_{x\to a} f(x)^{g(x)} = e^{\left(\lim_{x\to a} g(x)\cdot \ln f(x)\right)} = e^L$$ Apply L'Hôpital's rule to the exponent.	Same prescription as above.

Determinant Forms

There are no need to fix determinant forms:

Form	Limit
$\frac{A}{L}$	$\frac{A}{L}$
$\frac{0}{L}$	0
$\frac{\pm\infty}{L}$	$sign(L) \cdot \pm\infty$
$\frac{L}{0}$	$sign(L) \cdot \pm\infty$
$\frac{L}{\pm\infty}$	0

where A and L are constants not equal to zero and

$$sign(L) = \begin{cases} 1, \text{ if } L \geq 0; \\ -1, \text{ if } L < 0. \end{cases}$$