Math 212

Hypothetical Exam 2, (Chapter 3 in Thomas, Weir, Hass, and Giordano, 11th Ed.)
January 4, 1643
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Show all your work to receive credit. All answers must be justified to get full credit.

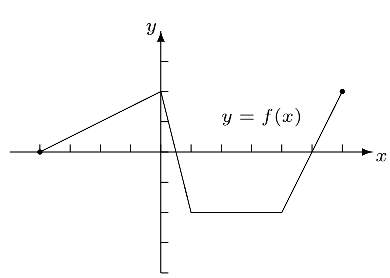
These questions are intended to give students in Math 212 some idea of the types of questions which could be asked on an exam. The questions may not cover all of the topics which will be on your exam (and they may cover more topics than are on your exam). The length of your exam may be shorter than this practice exam. **Working these problems is not a substitute for studying your notes, reading the book, or doing homework problems.**

Show Your Work

Show all work clearly and neatly. No work shown means no credit will be given. Use correct notation to get full credit. Reserve scratch paper work for scratch paper, which means only include necessary work on the exam. Erase all mistakes neatly. Keep it neat!

1. Find $\frac{dy}{dx}$ of $y = x^2 + 4x + 3$ using the definition of the derivative.

2. Given the following graph, answer the following questions. Each tick mark represents 1 unit.



(a) For which x-values does the derivative, $f'(x)$, not exist?

(b) Which points on the graph of $f(x)$ are continuous?

3. Find the derivative of the following functions:

   (a) $r = \left(\dfrac{\sin\theta}{\cos\theta - 1}\right)^2$

   (b) $y = x^3 + 2x^4 + 3x$

(c) $s = (\sec t + \tan t)^5$

(d) $y = \dfrac{\sqrt{t}}{1 + \sqrt{t}}$

4. Sand falls from a conveyor belt at the rate of 20 m^3/min onto the top of a cononical pile. The height of the pile is always three-eighths of the base diameter. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Answer the following in cm per minute.

   (a) How fast is the height of the pile changing when the pile is 4 m high?

   (b) How fast is the radius changing when the pile is 4 m high?

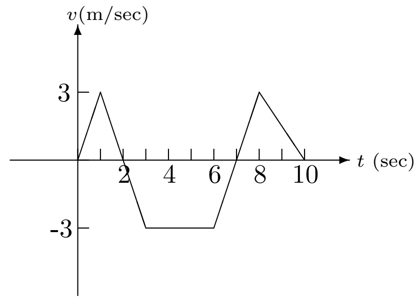
5. Given $f(x) = (1+x)^k$,

   (a) Show that the tangent line to $f(x)$ at $x = 0$ is
   $$y = 1 + kx.$$

   (b) Estimate $(1.00006)^{55}$ using the tangent line to $f(x)$. Compare to $(1.00006)^{55}$

6. Find y'' if $y = \sec x$. Express your answer using only $\sec x$ and $\tan x$.

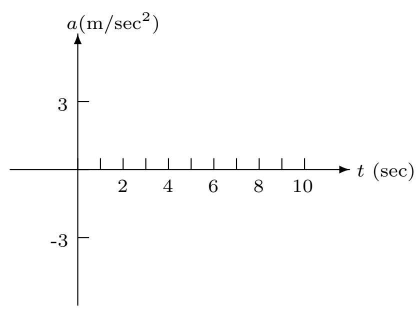
7. The accompanying figure shows the velocity $v = \frac{ds}{dt} = f(t)$ (m/sec) of a body moving along a coordinate line.



(a) When does the body reverse direction?

(b) When is the body moving at constant speed?

(c) Where defined, graph the acceleration of the body.



8. Derive the formula for the derivative of $\tan x$ by using the quotient rule.

9. Find an equation for the line tangent to the curve at the point defined by $t = \frac{\pi}{4}$, for the parametric equation

$$x = 2\cos t,\ \ y = 4\sin t.$$

   In addition, find the value of $\dfrac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

10. Use implicit differentiation to find $\frac{dy}{dx}$ for

$$y \sin\left(\frac{1}{y}\right) = 1 - xy.$$