Math 212

Hypothetical Exam 5, (Chapter 6 in Thomas, Weir, Hass, and Giordano, 11th Ed.)
July 16, 1945
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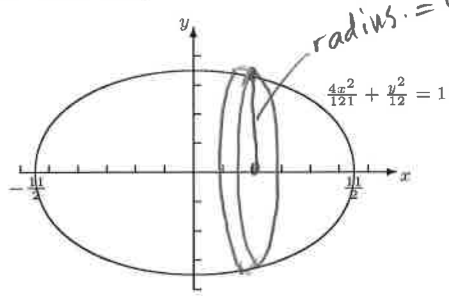
Name: Solution

Show all your work to receive credit. All answers must be justified to get full credit.

These questions are intended to give students in Math 212 some idea of the types of questions which could be asked on an exam. The questions may not cover all of the topics which will be on your exam (and they may cover more topics than are on your exam). The length of your exam may be shorter than this practice exam. **Working these problems is not a substitute for studying your notes, reading the book, or doing homework problems.**

Show Your Work
Show all work clearly and neatly. No work shown means no credit will be given. Use correct notation to get full credit. Reserve scratch paper work for scratch paper, which means only include necessary work on the exam. Erase all mistakes neatly. Keep it neat!

1. The profile of a football resembles the ellipse $\frac{4x^2}{121} + \frac{y^2}{12} = 1$, where x and y are measured in inches. Find the football's volume to the nearest cubic inch.



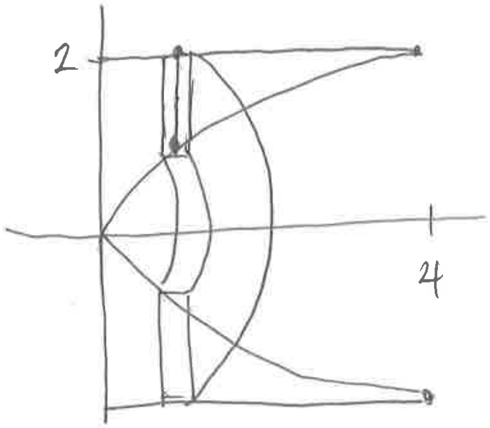
$\Rightarrow \frac{48x^2}{121} + y^2 = 12$

$$V = \int_{-11/2}^{11/2} \pi r^2\,dx = 2\int_0^{11/2} \pi r^2\,dx = 2\pi \int_0^{\frac{11}{2}} 12 - \frac{48x^2}{121}\,dx$$

$$= 2\pi\left[12x - \frac{16}{121}x^3\right]_0^{11/2} = 2\pi\left[12\left(\frac{11}{2}\right) - \frac{16}{121}\left(\frac{11}{2}\right)^3\right]$$

$$= \boxed{88\pi}$$

2. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the line $x = 4$.



$$V = \int_0^4 \pi (R^2 - r^2)\, dx$$

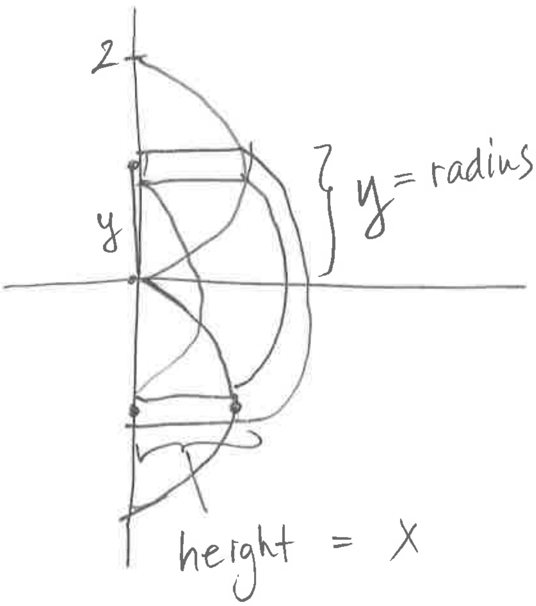
$$= \int_0^4 \pi (2^2 - y^2)\, dx$$

$$= \int_0^4 \pi (4 - (\sqrt{x})^2)\, dx$$

$$= \pi \int_0^4 4 - x \; dx = \pi \left(4x - \frac{x^2}{2}\right)_0^4$$

$$= \pi \left[16 - 8\right] = \boxed{8\pi}$$

3. Use the shell method to find the volume of the solid generated by revolving the region bounded by the curves $x = 2y - y^2$ and $x = 0$ about the x-axis.



$$V = \int 2\pi r h \, dy$$

$$= \int_0^2 2\pi y x \, dy$$

$$= \int_0^2 2\pi y (2y - y^2) \, dy$$

$$= 2\pi \int_0^2 2y^2 - y^3 \, dy$$

$$= 2\pi \left[\frac{2}{3} y^3 - \frac{y^4}{4} \right]_0^2$$

$$= 2\pi \left[\frac{16}{3} - 4 \right]$$

$$= \boxed{\frac{8}{3}\pi}$$

4. Find the length of the parametric curve $x = 1 - t$, $y = 2 + 3t$, from $-2/3 \leqslant t \leqslant 1$.

$$\frac{dx}{dt} = -1 \qquad \frac{dy}{dt} = 3$$

$$L = \int_{-\frac{2}{3}}^{1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_{-2/3}^{1} \sqrt{1+9}\, dt = \sqrt{10} \int_{-2/3}^{1} dt = \sqrt{10}\left(1 - \left(-\frac{2}{3}\right)\right)$$

$$= \sqrt{10}\left(\frac{5}{3}\right)$$

$$= \boxed{\frac{5\sqrt{10}}{3}}$$

I won't ask questions about of a parametric curve

Note: $x = 1 - t$

$\Rightarrow t = 1 - x$

$$y = 2 + 3t$$
$$= 2 + 3(1-x)$$
$$= 2 + 3 - 3x$$
$$\boxed{y = -3x + 5}$$

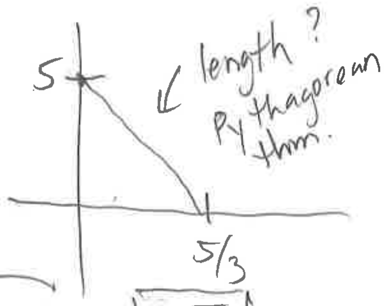
This is a line.

It connects between

$x = 1 - \left(-\frac{2}{3}\right) = \frac{5}{3}$ and $x = 1 - 1 = 0$

$y = 2 + 3\left(-\frac{2}{3}\right) = 0$ $\qquad$ $y = 2 + 3(1) = 5$

$\Rightarrow$



That's $\sqrt{5^2 + \left(\frac{5}{3}\right)^2} = \boxed{\frac{5\sqrt{10}}{3}}$

5. Find the center of mass of a thin plate covering the region bounded below by the parabola $y = x^2$ and above by the line $y = x$ if the plate's density at the point (x, y) is $\delta(x) = 12x$.

Not covered.

6. Find the area of the surface generated by revolving the curve $x = \sqrt{2y-1}$, $\frac{5}{8} \leqslant y \leqslant 1$, about the y=axis.

Not covered

7. A mountain climber is about to haul up a $50m$ length of hanging rope. How much work will it take if the rop weighs $0.624\ N/m$?

I won't ask work problems on this exam

8. A rectangular milk carton measures 3.75 in. $\times$ 3.75 in. at the base and is 7.75 in. tall. Find the force of the milk on one side when the carton is full. Assume milk has the weight density of $64.5 lb/ft^3$.

won't ask a question
about force.

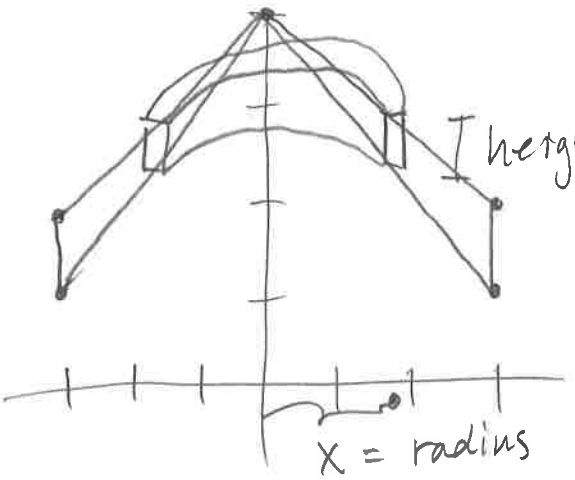
9. Find the length of the curve $y = x^{1/2} - \frac{1}{3}x^{3/2}$ from $x = 1$ to $x = 4$.

Won't ask on this exam

about arclength

10. Find the volume of the region bounded by $y = \overset{y_1 \downarrow}{-x + 4}$, $y = \overset{y_2 \downarrow}{-\frac{2}{3}x + 4}$, and $x = 3$ rotated about the y-axis.

Shell method is easiest



height $= y_2 - y_1$

So $$V = \int_0^3 2\pi r h \, dx = \int_0^3 2\pi x (y_2 - y_1) \, dx$$

$$= \int_0^3 2\pi x \underbrace{\left(-\tfrac{2}{3}x + 4 - (-x + 4)\right)}_{x/3} dx$$

$$= \frac{2}{3}\pi \int_0^3 x^2 \, dx = \frac{2}{3}\pi \left[\frac{x^3}{3}\right]_0^3 = \frac{2}{3}\pi\left(\frac{27}{3}\right)$$

$$= \boxed{6\pi}$$