

A Transition to Advanced Mathematics, 6th Ed

Read Preface!

1.1

Propositions and Connectives

DEF: A proposition is a sentence that is either true or false

These are propositions

Ex: a) $98) - 42$ is not a positive number

b) $1 + 1 = 5$

c) the elephant will become extinct in 2525

d) Julius ceasar had two eggs for breakfast on his tenth birthday.

These aren't propositions

e) what did you say? (Interrogatory, no truth value)

f) $x^2 = 36$ (depends on what x is)

g) she has your car keys (depends on who she is)

h) This sentence is false

paradox: a statement that is neither true nor false.

Simple or atomic propositions - a prop that does not have the props of comp. as part of it.

compound prop. propositions formed by using logical connectives with simple propos.

DEF: Given propositions P and Q

Conjunction - $P \wedge Q$ "P and Q"
True ^{exactly} when both P and Q are true

disjunction - $P \vee Q$ "P or Q"
True exactly when at least one of P or Q is true.

negation - $\sim P$ "not P"
True when P is false.

Examples

$P = "1 \neq 3"$

Q is "7 is odd"

P is true, Q is true, so

$P \wedge Q$ is true

$P \vee Q$ is true

$\sim P$ is false ($\sim P = "1 = 3"$)

Ex: $\underbrace{\sqrt{2} < \sqrt{3}}_P$ or $\underbrace{\text{chickens have lips}}_Q$

P is true

Q is false

So $\sqrt{2} < \sqrt{3}$ or chickens have lips is true.

The props $\sim(P \wedge Q)$ and $\sim P \vee \sim Q$
(one of De Morgan's Laws) To show this, use
Truth Tables.

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T



The same!

You don't have both P and $Q \rightarrow \sim(P \wedge Q)$

same as

You don't have P or you don't have Q

DEF: A tautology is a propositional form that is true for every assignment of truth values to its components

Ex: $P \vee \sim P$

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

Ex: $(P \vee Q) \vee (\sim P \wedge \sim Q)$

P	Q	$P \vee Q$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	
T	T	T	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	F	T	T	T	T

DEF: A contradiction is a propositional form that is false for every assignment of truth values to its components.

A contradiction is the negation of a tautology!

DEF: A denial of a prop P is any prop equivalent to $\sim P$.

○ Ex: $P = \text{"}\sqrt{2} \text{ is irrational"}$

Denials:

It is not the case that $\sqrt{2}$ is irrational (the negation)

$\sqrt{2}$ is the quotient of two integers.

The decimal expansion of $\sqrt{2}$ is repeating or terminating.

It is not the case that it is not the case that $\sqrt{2}$ is rational.

Ambiguities

○ Use delimiters to avoid ambiguities:

delimiters:	()	parentheses
	[]	square brackets
	{ }	braces (curly)

To avoid using too many delimiters, use these rules:

$\sim$	applies to the smallest	proportion following it
$\wedge$	connects the smallest	" surrounding it
$\vee$	"	"
	"	"
	"	"
	"	"

○ Ex:

$$\sim P \vee Q = (\sim P) \vee Q$$

$$P \wedge \sim Q \vee \sim R = [P \wedge (\sim Q)] \vee (\sim R)$$