

2.2 set operations

Binary operations on sets = use two sets

DEF: Let A and B be sets

The union of A and B is the set $A \cup B = \{x: x \in A \text{ or } x \in B\}$

The intersection of A and B is the set $A \cap B = \{x: x \in A \text{ and } x \in B\}$

The difference of A and B is the set $A - B = \{x: x \in A \text{ and } x \notin B\}$

Ex: $A = \{1, 2, 3, 5\}$ $B = \{2, 3, 7, 9\}$

$$A \cap B = \{2, 3\}$$

$$A \cup B = \{1, 2, 3, 5, 7, 9\}$$

$$A - B = \{1, 5\}$$

DEF: Two sets A and B are disjoint iff $A \cap B = \emptyset$.

Thm: Let A, B, C be sets.

$$(a) A \subseteq A \cup B$$

$$(b) A \cap B \subseteq A$$

$$(c) A \cap \emptyset = \emptyset$$

$$(d) A \cup \emptyset = A$$

$$(e) A \cap A = A$$

$$(f) A \cup A = A$$

$$(g) A \cup B = B \cup A$$

$$(h) A \cap B = B \cap A$$

$$(i) A - \emptyset = A$$

$$(j) \emptyset - A = \emptyset$$

$$(k) A \cup (B \cap C) = (A \cup B) \cap C$$

$$(l) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(m) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(n) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$(o) A \subseteq B \text{ iff } A \cup B = B$$

$$(p) A \subseteq B \text{ iff } A \cap B = A$$

$$(q) \text{ If } A \subseteq B, \text{ then } A \cup C \subseteq B \cup C$$

$$(r) \text{ If } A \subseteq B, \text{ then } A \cap C \subseteq B \cap C$$

DEF: If U is the universe and $B \subseteq U$, then the complement of B is the set $\tilde{B} = U - B$

$\tilde{B}$ is all elements of the universe that are not in B .

Thm 2.7

Let U be the universe, and let A, B be subsets of U . Then

(a) $\tilde{\tilde{A}} = A$

(b) $A \cup \tilde{A} = U$

(c) $A \cap \tilde{A} = \emptyset$

(d) $A - B = A \cap \tilde{B}$

(e) $A \subseteq B$ iff $\tilde{B} \subseteq \tilde{A}$

(f) $\widetilde{A \cup B} = \tilde{A} \cap \tilde{B}$

(g) $\widetilde{A \cap B} = \tilde{A} \cup \tilde{B}$

(h) $A \cap B = \emptyset$ iff $A \subseteq \tilde{B}$.

} De Morgan's Laws.