

4.4 Images of Sets

DEF: let $f: A \rightarrow B$ and $X \subseteq A, Y \subseteq B$.

The image of X or image set of X is

$$f(X) = \{y \in B : y = f(x) \text{ for some } x \in X\}$$

and the inverse image of Y is

$$f^{-1}(Y) = \{x \in A : f(x) \in Y\}$$

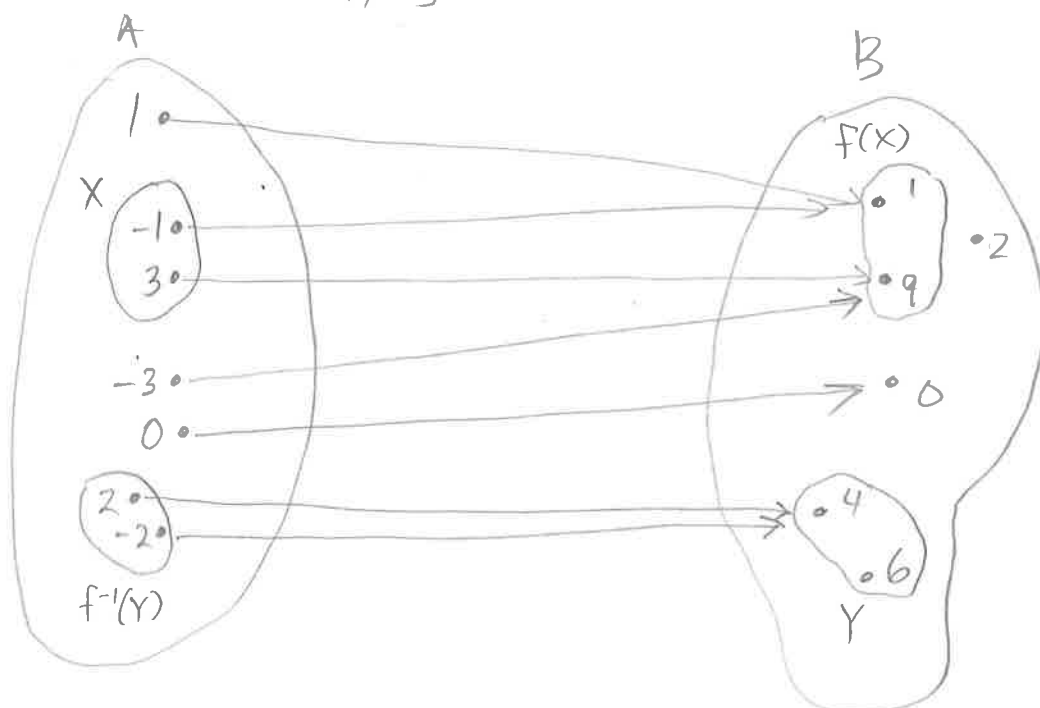
EX: let $A = \{0, 1, 2, 3, -1, -2, -3\}$

$$B = \{0, 1, 2, 4, 6, 9\}$$

$$X = \{-1, 3\}$$

$$Y = \{4, 6\}$$

$$f: A \rightarrow B \quad f(x) = x^2$$



$$f(X) = f(\{-1, 3\}) = \{1, 9\}$$

$$f(\{-3, 3\}) = \{9\}$$

$$f(A) = \{0, 1, 4, 9\}$$

$$f^{-1}(Y) = f^{-1}(\{4, 6\})$$

$$= \{-2, 2\}$$

$$f^{-1}(\{6\}) = \emptyset$$

$$f^{-1}(B) = A$$

Thm

Let $f: A \rightarrow B$

C, D subsets of A

E, F subsets of B .

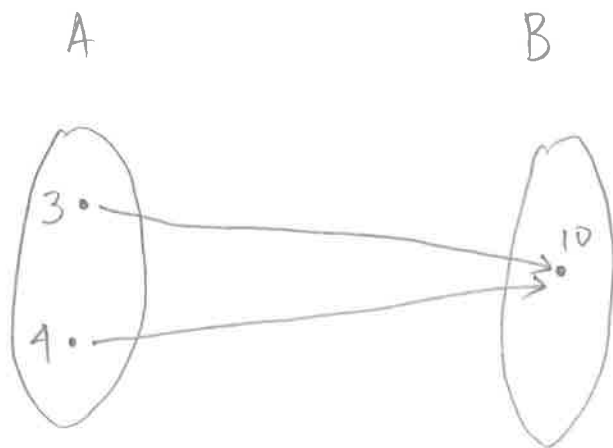
$$(a) \quad f(C \cap D) \subseteq f(C) \cap f(D)$$

$$(b) \quad f(C \cup D) = f(C) \cup f(D)$$

$$(c) \quad f^{-1}(E \cap F) = f^{-1}(E) \cap f^{-1}(F)$$

$$(d) \quad f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$$

Why is (a) $\subseteq$?



$$\text{Let } C = \{3\}$$

$$D = \{4\}$$

$$\text{Then } C \cap D = \emptyset$$

$$\text{and } f(C \cap D) = \emptyset$$

$$\begin{aligned} f(C) \cap f(D) &= \{10\} \cap \{10\} \\ &= \{10\} \end{aligned}$$

$$\text{So } \emptyset \subseteq \{10\}.$$

Note: f^{-1} is not a function from B to A

so it doesn't make sense to consider $f^{-1}(1)$

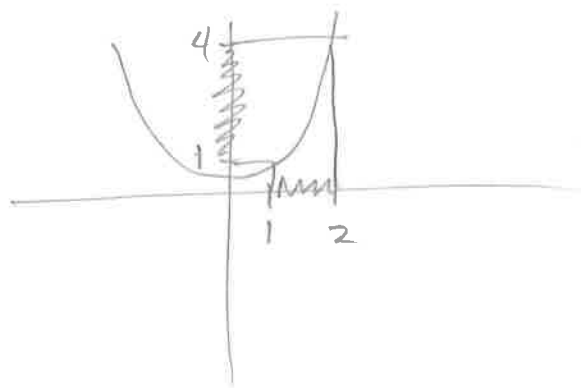
However, $f^{-1}(\{1\})$ is meaningful and is equal to $\{-1, 1\}$

Ex: $f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = x^2$

$$f(\{-2, 2\}) = \{4\}$$

$$f([1, 2]) = [1, 4]$$

$$f([-1, 2]) = [0, 4]$$



$$f^{-1}([1, 4]) = [-2, -1] \cup [1, 2]$$

