Introduction to R Programming / Rstudio and R Markdown / R Notebooks

Scott K. Hyde
Mathematics Department

Brigham Young University – Hawaii
Laie, Hawaii

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Outline

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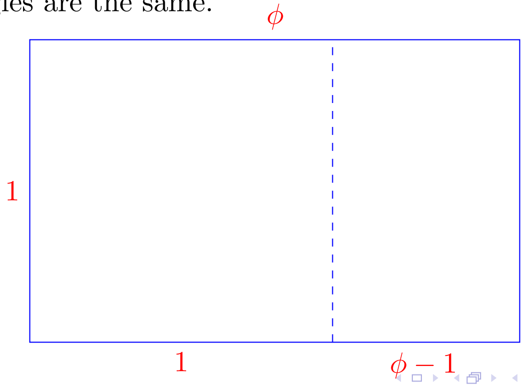

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The positive solution to the equation is

$$\phi = \frac{1 + \sqrt{5}}{2}$$

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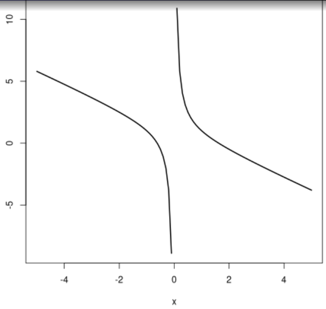

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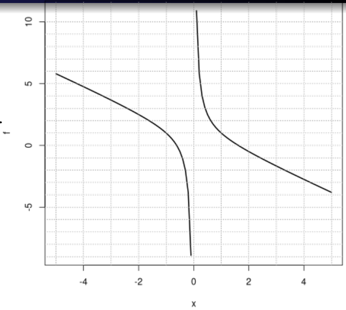

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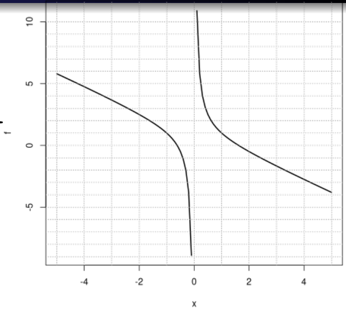

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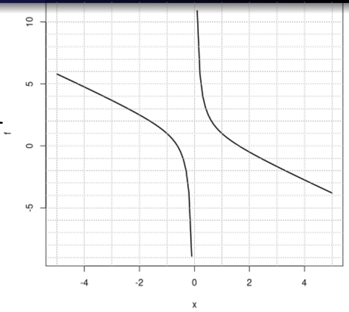

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- Type `help(plot)` for more options.
- There are other commands like `curve`

Fibonacci Numbers

Fibonacci Numbers appear often in nature, such as petals of flowers, organization of sunflower seeds, pine cones, branches on trees, shell spirals, reproduction, and even more.

Leonardo Fibonacci originally posed the following question:

> *A newly born breeding pair of rabbits are put in a field; each breeding pair mates at the age of one month, and at the end of their second month, they always produce another pair of rabbits; and rabbits never die, but continue breeding forever. Fibonacci posed the puzzle: how many pairs will there be in one year?*

Fibonacci's problem introduces a delay for maturation, which made the problem more complicated.

Fibonacci Numbers

If f_n dentoes the number of pairs of rabbits after n months, then the number of pairs is the number at the beginning plus the number of births:

$$f_n = f_{n-1} + f_{n-2}$$

with initial conditions of $f_0 = f_1 = 1$. So we see

$$\begin{aligned} f_2 &= f_1 + f_0 = 1 + 1 = 2 \\ f_3 &= f_2 + f_1 = 2 + 1 = 3 \\ f_4 &= f_3 + f_2 = 3 + 2 = 5 \\ &\dots \end{aligned}$$

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```
> fib(12)
[1]   1   2   3   5   8  13  21  34  55  89 144 233
```

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```
fibmax = function(M) {
  ## Fibonacci Maximum - returns months and
  ## fibonaccinumbers until supasses M.
  m = c(1,2) #start with 1 and 2
  k=3
  while (m[k-1] < M) {
    m = c(m,m[k-1] + m[k-2])
    k = k + 1
  }
  list(sequence=m,months=length(m))
}
```

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While loops are valuable for stopping a loop when a criterion has been met.

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Here's a few of the ratios:

$$f_6/f_5 = 1.625$$
$$f_{26}/f_{25} = 1.61803398878024$$
$$f_{40}/f_{39} = 1.61803398874989$$

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There are limitations to computer arithmetic so, we will have roundoff error if an element is too big.

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```
collatz = function(n) { ## Show the 3n+1 sequence.
  s = n #start with n
  while (n > 1) {
    if ( n %% 2 == 0) {
      n=n/2
    } else {
      n = 3*n+1
    }
    s=c(s,n) #Add n to the s vector
  }
  s
}
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- Try and find the longest sequence for numbers from 1-100!
- I wrote a program and found the longest from 1–10,000. It is 6171 with a length of 262! Try 27!

Bessel Functions

Some functions are defined in terms of an infinite sum. For example, the Bessel Function is a solution to Bessel's Equation:

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- To find the solution to this, take Differential Equations.
- A solution to the equation is the m^{th} order Bessel function of the 1^{st} kind (m is an integer). Its series is

$$J_m(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(m+n)!}\left(\frac{x}{2}\right)^{m+2n}$$

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- Use a while command to iterate and build the sum.
- If the new term is larger than the tolerance, then repeat.
- If it is smaller, then quit.
- The input to the function is a vector, so it will evaluate the function at each value in the vector.

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Dividing t_{n+1} by t_n will help develop the recursion. So

$$\frac{t_{n+1}}{t_n} = \frac{\frac{(-1)^{n+1}}{(n+1)!(m+n+1)!}\left(\frac{x}{2}\right)^{m+2n+2}}{\frac{(-1)^n}{n!(m+n)!}\left(\frac{x}{2}\right)^{m+2n}} = \frac{-1}{(n+1)(m+n+1)}\left(\frac{x}{2}\right)^2$$

Bessel Functions

The n^{th} and $(n+1)^{\text{th}}$ terms are

$$t_n = \frac{(-1)^n}{n!(m+n)!}\left(\frac{x}{2}\right)^{m+2n} \qquad t_{n+1} = \frac{(-1)^{n+1}}{(n+1)!(m+n+1)!}\left(\frac{x}{2}\right)^{m+2(n+1)}$$

Dividing t_{n+1} by t_n will help develop the recursion. So

$$\frac{t_{n+1}}{t_n} = \frac{\frac{(-1)^{n+1}}{(n+1)!(m+n+1)!}\left(\frac{x}{2}\right)^{m+2n+2}}{\frac{(-1)^n}{n!(m+n)!}\left(\frac{x}{2}\right)^{m+2n}} = \frac{-1}{(n+1)(m+n+1)}\left(\frac{x}{2}\right)^2$$

So

$$t_{n+1} = \frac{-x^2}{4(n+1)(m+n+1)}t_n$$

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```
bessel = function(x,m,tol=1e-10) {
  ## Approx to J_m(x) to a given tolerance
  k=1
  summa = term = (x/2)^m/factorial(m)
  while (max(abs(term)) >= tol) {
    term = term*(-x^2)/(4*k*(k+m))
    summa = summa + term;
    k=k+1
  }
  summa
}
```

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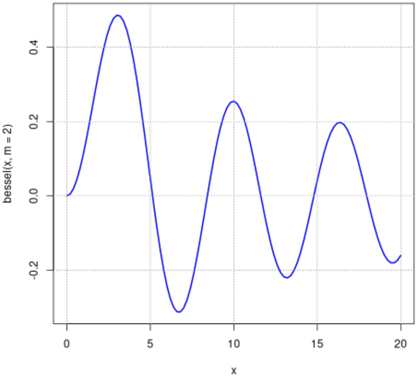

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> colors=c("blue","green","red","purple","orange")
> x=seq(0,20,length=1000)
> plot(x,bessel(x,m=0),type="l",col=colors[1],lwd=2,
   ylab=expression(J[m](x)),main=expression(
  paste("Bessel Functions ",J[0](x)," to ",J[4](x))))
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> for (i in 1:4)
  points(x,bessel(x,m=i),type="l",col=colors[i+1],lwd=2)
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> abline(h=0.2*(-2:5),v=5*(0:4),lty=3,col="gray")
> legend(15,1,legend=c(expression(J[0](x),J[1](x),J[2](x),
    J[3](x),J[4](x))),col=colors,lty=1,lwd=2)
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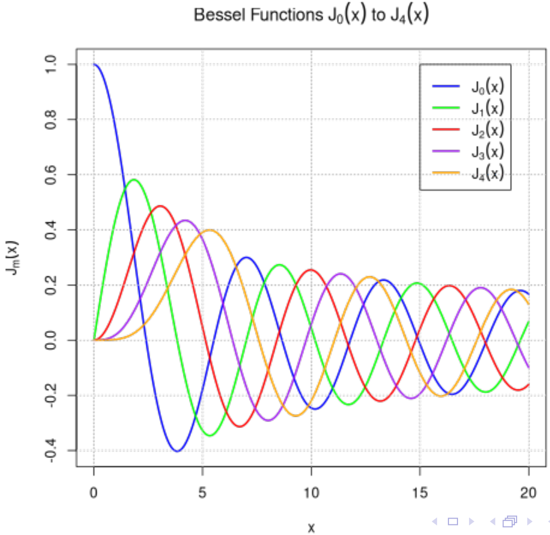

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- The markup language is called “R-Markdown”.
- In our class, you will be required to create your problem set solutions using “R-Notebooks” (or R-Markdown).

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- You can also go to the link which has the output from this presentation in R-Notebook mode. Click on it and learn!