Interpolation & Polynomial Approximation

Lagrange Interpolating Polynomials I

Numerical Analysis (9th Edition)

R L Burden & J D Faires

Beamer Presentation Slides
prepared by
John Carroll
Dublin City University

Outline

Outline

1. Weierstrass Approximation Theorem
2. Inaccuracy of Taylor Polynomials

Outline

Outline

Outline

Weierstrass Approximation Theorem

Algebraic Polynomials

Weierstrass Approximation Theorem

Algebraic Polynomials

One of the most useful and well-known classes of functions mapping the set of real numbers into itself is the algebraic polynomials, the set of functions of the form

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is a nonnegative integer and $a_0, \ldots, a_n$ are real constants.

Weierstrass Approximation Theorem

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

Algebraic Polynomials (Cont'd)

Weierstrass Approximation Theorem

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

Algebraic Polynomials (Cont'd)

- One reason for their importance is that they uniformly approximate continuous functions.

Weierstrass Approximation Theorem

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

Algebraic Polynomials (Cont'd)

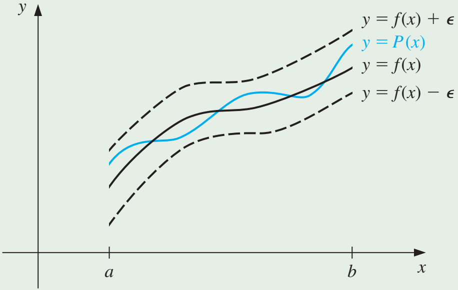
- One reason for their importance is that they uniformly approximate continuous functions.
- By this we mean that given any function, defined and continuous on a closed and bounded interval, there exists a polynomial that is as "close" to the given function as desired.

Weierstrass Approximation Theorem

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

Algebraic Polynomials (Cont'd)

- One reason for their importance is that they uniformly approximate continuous functions.
- By this we mean that given any function, defined and continuous on a closed and bounded interval, there exists a polynomial that is as "close" to the given function as desired.
- This result is expressed precisely in the Weierstrass Approximation Theorem.



Weierstrass Approximation Theorem

Suppose that f is defined and continuous on $[a, b]$. For each $\epsilon > 0$, there exists a polynomial $P(x)$, with the property that

$$|f(x) - P(x)| < \epsilon, \quad \text{for all } x \text{ in } [a, b].$$

Benefits of Algebraic Polynomials

Benefits of Algebraic Polynomials

- Another important reason for considering the class of polynomials in the approximation of functions is that the derivative and indefinite integral of a polynomial are easy to determine and are also polynomials.

Benefits of Algebraic Polynomials

- Another important reason for considering the class of polynomials in the approximation of functions is that the derivative and indefinite integral of a polynomial are easy to determine and are also polynomials.
- For these reasons, polynomials are often used for approximating continuous functions.

Outline

The Lagrange Polynomial: Taylor Polynomials

Interpolating with Taylor Polynomials

The Lagrange Polynomial: Taylor Polynomials

Interpolating with Taylor Polynomials

- The Taylor polynomials are described as one of the fundamental building blocks of numerical analysis.

The Lagrange Polynomial: Taylor Polynomials

Interpolating with Taylor Polynomials

- The Taylor polynomials are described as one of the fundamental building blocks of numerical analysis.
- Given this prominence, you might expect that polynomial interpolation would make heavy use of these functions.

The Lagrange Polynomial: Taylor Polynomials

Interpolating with Taylor Polynomials

- The Taylor polynomials are described as one of the fundamental building blocks of numerical analysis.
- Given this prominence, you might expect that polynomial interpolation would make heavy use of these functions.
- However this is not the case.

The Lagrange Polynomial: Taylor Polynomials

Interpolating with Taylor Polynomials

- The Taylor polynomials are described as one of the fundamental building blocks of numerical analysis.
- Given this prominence, you might expect that polynomial interpolation would make heavy use of these functions.
- However this is not the case.
- The Taylor polynomials agree as closely as possible with a given function at a specific point, but they concentrate their accuracy near that point.

The Lagrange Polynomial: Taylor Polynomials

Interpolating with Taylor Polynomials

- The Taylor polynomials are described as one of the fundamental building blocks of numerical analysis.
- Given this prominence, you might expect that polynomial interpolation would make heavy use of these functions.
- However this is not the case.
- The Taylor polynomials agree as closely as possible with a given function at a specific point, but they concentrate their accuracy near that point.
- A good interpolation polynomial needs to provide a relatively accurate approximation over an entire interval, and Taylor polynomials do not generally do this.

The Lagrange Polynomial: Taylor Polynomials

Example: $f(x) = e^x$

We will calculate the first six Taylor polynomials about $x_0 = 0$ for $f(x) = e^x$.

The Lagrange Polynomial: Taylor Polynomials

Example: $f(x) = e^x$

We will calculate the first six Taylor polynomials about $x_0 = 0$ for $f(x) = e^x$.

Note

Since the derivatives of $f(x)$ are all e^x, which evaluated at $x_0 = 0$ gives 1.

The Lagrange Polynomial: Taylor Polynomials

Example: $f(x) = e^x$

We will calculate the first six Taylor polynomials about $x_0 = 0$ for $f(x) = e^x$.

Note

Since the derivatives of $f(x)$ are all e^x, which evaluated at $x_0 = 0$ gives 1.

The Taylor polynomials are as follows:

Taylor Polynomials for $f(x) = e^x$ about $x_0 = 0$

$$P_0(x) = 1$$

$$P_1(x) = 1 + x$$

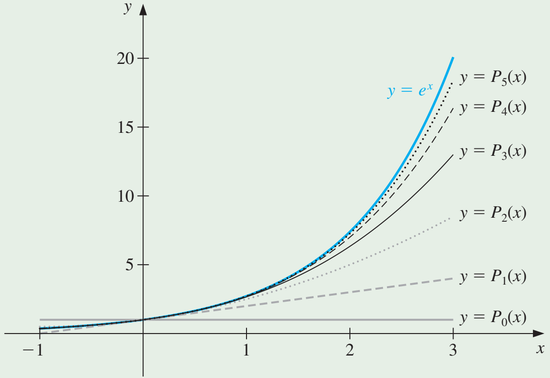
$$P_2(x) = 1 + x + \frac{x^2}{2}$$

$$P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

$$P_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$$

$$P_5(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$$

Taylor Polynomials for $f(x) = e^x$ about $x_0 = 0$



Notice that even for the higher-degree polynomials, the error becomes progressively worse as we move away from zero.

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

Example: A more extreme case

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

Example: A more extreme case

- Although better approximations are obtained for $f(x) = e^x$ if higher-degree Taylor polynomials are used, this is not true for all functions.

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

Example: A more extreme case

- Although better approximations are obtained for $f(x) = e^x$ if higher-degree Taylor polynomials are used, this is not true for all functions.
- Consider, as an extreme example, using Taylor polynomials of various degrees for $f(x) = \frac{1}{x}$ expanded about $x_0 = 1$ to approximate $f(3) = \frac{1}{3}$.

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

Calculations

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

Calculations

Since

$$f(x) = x^{-1},\ f'(x) = -x^{-2},\ f''(x) = (-1)^2 2 \cdot x^{-3},$$

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

Calculations

Since

$$f(x) = x^{-1},\ f'(x) = -x^{-2},\ f''(x) = (-1)^2 2 \cdot x^{-3},$$

and, in general,

$$f^{(k)}(x) = (-1)^k k! x^{-k-1},$$

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

Calculations

Since

$$f(x) = x^{-1},\ f'(x) = -x^{-2},\ f''(x) = (-1)^2 2 \cdot x^{-3},$$

and, in general,

$$f^{(k)}(x) = (-1)^k k! x^{-k-1},$$

the Taylor polynomials are

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(1)}{k!}(x-1)^k = \sum_{k=0}^{n}(-1)^k(x-1)^k.$$

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

To Approximate $f(3) = \frac{1}{3}$ by $P_n(3)$

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

To Approximate $f(3) = \frac{1}{3}$ by $P_n(3)$

- To approximate $f(3) = \frac{1}{3}$ by $P_n(3)$ for increasing values of n, we obtain the values shown below — rather a dramatic failure!

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

To Approximate $f(3) = \frac{1}{3}$ by $P_n(3)$

- To approximate $f(3) = \frac{1}{3}$ by $P_n(3)$ for increasing values of n, we obtain the values shown below — rather a dramatic failure!
- When we approximate $f(3) = \frac{1}{3}$ by $P_n(3)$ for larger values of n, the approximations become increasingly inaccurate.

Taylor Polynomials for $f(x) = \frac{1}{x}$ about $x_0 = 1$

To Approximate $f(3) = \frac{1}{3}$ by $P_n(3)$

- To approximate $f(3) = \frac{1}{3}$ by $P_n(3)$ for increasing values of n, we obtain the values shown below — rather a dramatic failure!
- When we approximate $f(3) = \frac{1}{3}$ by $P_n(3)$ for larger values of n, the approximations become increasingly inaccurate.

n	0	1	2	3	4	5	6	7
$P_n(3)$	1	-1	3	-5	11	-21	43	-85

The Lagrange Polynomial: Taylor Polynomials

Footnotes

The Lagrange Polynomial: Taylor Polynomials

Footnotes

- For the Taylor polynomials, all the information used in the approximation is concentrated at the single number x_0, so these polynomials will generally give inaccurate approximations as we move away from x_0.

The Lagrange Polynomial: Taylor Polynomials

Footnotes

- For the Taylor polynomials, all the information used in the approximation is concentrated at the single number x_0, so these polynomials will generally give inaccurate approximations as we move away from x_0.
- This limits Taylor polynomial approximation to the situation in which approximations are needed only at numbers close to x_0.

The Lagrange Polynomial: Taylor Polynomials

Footnotes

- For the Taylor polynomials, all the information used in the approximation is concentrated at the single number x_0, so these polynomials will generally give inaccurate approximations as we move away from x_0.
- This limits Taylor polynomial approximation to the situation in which approximations are needed only at numbers close to x_0.
- For ordinary computational purposes, it is more efficient to use methods that include information at various points.

The Lagrange Polynomial: Taylor Polynomials

Footnotes

- For the Taylor polynomials, all the information used in the approximation is concentrated at the single number x_0, so these polynomials will generally give inaccurate approximations as we move away from x_0.
- This limits Taylor polynomial approximation to the situation in which approximations are needed only at numbers close to x_0.
- For ordinary computational purposes, it is more efficient to use methods that include information at various points.
- The primary use of Taylor polynomials in numerical analysis is not for approximation purposes, but for the derivation of numerical techniques and error estimation.

Outline

The Lagrange Polynomial: The Linear Case

Polynomial Interpolation

The Lagrange Polynomial: The Linear Case

Polynomial Interpolation

- The problem of determining a polynomial of degree one that passes through the distinct points

$$(x_0, y_0) \quad \text{and} \quad (x_1, y_1)$$

is the same as approximating a function f for which

$$f(x_0) = y_0 \quad \text{and} \quad f(x_1) = y_1$$

by means of a first-degree polynomial interpolating, or agreeing with, the values of f at the given points.

The Lagrange Polynomial: The Linear Case

Polynomial Interpolation

- The problem of determining a polynomial of degree one that passes through the distinct points

$$(x_0, y_0) \qquad \text{and} \qquad (x_1, y_1)$$

is the same as approximating a function f for which

$$f(x_0) = y_0 \qquad \text{and} \qquad f(x_1) = y_1$$

by means of a first-degree polynomial interpolating, or agreeing with, the values of f at the given points.

- Using this polynomial for approximation within the interval given by the endpoints is called polynomial interpolation.

The Lagrange Polynomial: The Linear Case

Define the functions

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} \quad \text{and} \quad L_1(x) = \frac{x - x_0}{x_1 - x_0}.$$

The Lagrange Polynomial: The Linear Case

Define the functions

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} \quad \text{and} \quad L_1(x) = \frac{x - x_0}{x_1 - x_0}.$$

Definition

The linear Lagrange interpolating polynomial though (x_0, y_0) and (x_1, y_1) is

$$P(x) = L_0(x)f(x_0) + L_1(x)f(x_1) = \frac{x - x_1}{x_0 - x_1}f(x_0) + \frac{x - x_0}{x_1 - x_0}f(x_1).$$

The Lagrange Polynomial: The Linear Case

$$P(x) = L_0(x)f(x_0) + L_1(x)f(x_1) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1).$$

Note that

$$L_0(x_0) = 1, \quad L_0(x_1) = 0, \quad L_1(x_0) = 0, \quad \text{and} \quad L_1(x_1) = 1,$$

The Lagrange Polynomial: The Linear Case

$$P(x) = L_0(x)f(x_0) + L_1(x)f(x_1) = \frac{x - x_1}{x_0 - x_1}f(x_0) + \frac{x - x_0}{x_1 - x_0}f(x_1).$$

Note that

$$L_0(x_0) = 1, \quad L_0(x_1) = 0, \quad L_1(x_0) = 0, \quad \text{and} \quad L_1(x_1) = 1,$$

which implies that

$$P(x_0) = 1 \cdot f(x_0) + 0 \cdot f(x_1) = f(x_0) = y_0$$

and

$$P(x_1) = 0 \cdot f(x_0) + 1 \cdot f(x_1) = f(x_1) = y_1.$$

The Lagrange Polynomial: The Linear Case

$$P(x) = L_0(x)f(x_0) + L_1(x)f(x_1) = \frac{x - x_1}{x_0 - x_1}f(x_0) + \frac{x - x_0}{x_1 - x_0}f(x_1).$$

Note that

$$L_0(x_0) = 1, \quad L_0(x_1) = 0, \quad L_1(x_0) = 0, \quad \text{and} \quad L_1(x_1) = 1,$$

which implies that

$$P(x_0) = 1 \cdot f(x_0) + 0 \cdot f(x_1) = f(x_0) = y_0$$

and

$$P(x_1) = 0 \cdot f(x_0) + 1 \cdot f(x_1) = f(x_1) = y_1.$$

So P is the unique polynomial of degree at most 1 that passes through (x_0, y_0) and (x_1, y_1).

The Lagrange Polynomial: The Linear Case

Example: Linear Interpolation

Determine the linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$.

The Lagrange Polynomial: The Linear Case

Example: Linear Interpolation

Determine the linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$.

Solution

In this case we have

$$L_0(x) = \frac{x-5}{2-5} = -\frac{1}{3}(x-5) \quad \text{and} \quad L_1(x) = \frac{x-2}{5-2} = \frac{1}{3}(x-2),$$

The Lagrange Polynomial: The Linear Case

Example: Linear Interpolation

Determine the linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$.

Solution

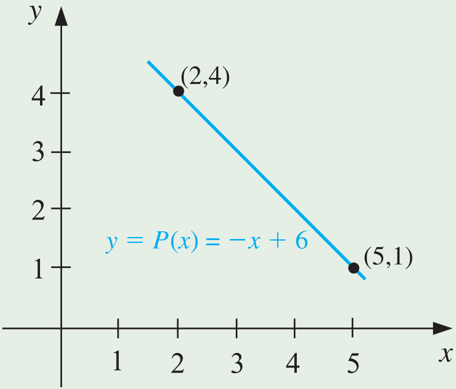
In this case we have

$$L_0(x) = \frac{x-5}{2-5} = -\frac{1}{3}(x-5) \quad \text{and} \quad L_1(x) = \frac{x-2}{5-2} = \frac{1}{3}(x-2),$$

so

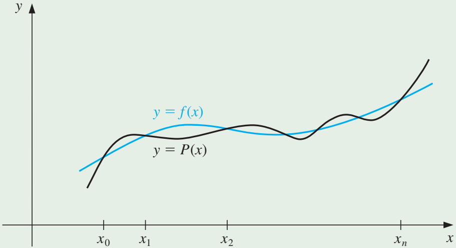
$$P(x) = -\frac{1}{3}(x-5)\cdot 4 + \frac{1}{3}(x-2)\cdot 1 = -\frac{4}{3}x + \frac{20}{3} + \frac{1}{3}x - \frac{2}{3} = -x + 6.$$

The Lagrange Polynomial: The Linear Case



The linear Lagrange interpolating polynomial that passes through the points $(2, 4)$ and $(5, 1)$.

The Lagrange Polynomial: Degree n Construction



To generalize the concept of linear interpolation, consider the construction of a polynomial of degree at most n that passes through the $n + 1$ points

$$(x_0, f(x_0)),\ (x_1, f(x_1)), \ldots, (x_n, f(x_n)).$$

The Lagrange Polynomial: The General Case

Constructing the Degree n Polynomial

The Lagrange Polynomial: The General Case

Constructing the Degree n Polynomial

- We first construct, for each $k = 0, 1, \ldots, n$, a function $L_{n,k}(x)$ with the property that $L_{n,k}(x_i) = 0$ when $i \neq k$ and $L_{n,k}(x_k) = 1$.

The Lagrange Polynomial: The General Case

Constructing the Degree n Polynomial

- We first construct, for each $k = 0, 1, \ldots, n$, a function $L_{n,k}(x)$ with the property that $L_{n,k}(x_i) = 0$ when $i \neq k$ and $L_{n,k}(x_k) = 1$.
- To satisfy $L_{n,k}(x_i) = 0$ for each $i \neq k$ requires that the numerator of $L_{n,k}(x)$ contain the term

$$(x - x_0)(x - x_1) \cdots (x - x_{k-1})(x - x_{k+1}) \cdots (x - x_n).$$

The Lagrange Polynomial: The General Case

Constructing the Degree *n* Polynomial

- We first construct, for each $k = 0, 1, \ldots, n$, a function $L_{n,k}(x)$ with the property that $L_{n,k}(x_i) = 0$ when $i \neq k$ and $L_{n,k}(x_k) = 1$.
- To satisfy $L_{n,k}(x_i) = 0$ for each $i \neq k$ requires that the numerator of $L_{n,k}(x)$ contain the term

$$(x - x_0)(x - x_1) \cdots (x - x_{k-1})(x - x_{k+1}) \cdots (x - x_n).$$

- To satisfy $L_{n,k}(x_k) = 1$, the denominator of $L_{n,k}(x)$ must be this same term but evaluated at $x = x_k$.

The Lagrange Polynomial: The General Case

Constructing the Degree n Polynomial

- We first construct, for each $k = 0, 1, \ldots, n$, a function $L_{n,k}(x)$ with the property that $L_{n,k}(x_i) = 0$ when $i \neq k$ and $L_{n,k}(x_k) = 1$.
- To satisfy $L_{n,k}(x_i) = 0$ for each $i \neq k$ requires that the numerator of $L_{n,k}(x)$ contain the term

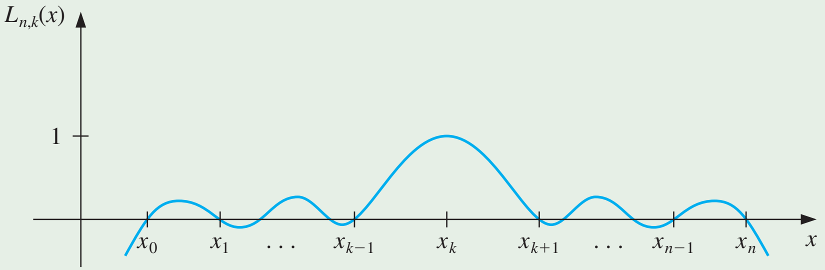
$$(x - x_0)(x - x_1) \cdots (x - x_{k-1})(x - x_{k+1}) \cdots (x - x_n).$$

- To satisfy $L_{n,k}(x_k) = 1$, the denominator of $L_{n,k}(x)$ must be this same term but evaluated at $x = x_k$.
- Thus

$$L_{n,k}(x) = \frac{(x - x_0) \cdots (x - x_{k-1})(x - x_{k+1}) \cdots (x - x_n)}{(x_k - x_0) \cdots (x_k - x_{k-1})(x_k - x_{k+1}) \cdots (x_k - x_n)}.$$

The Lagrange Polynomial: The General Case

$$L_{n,k}(x) = \frac{(x - x_0)\cdots(x - x_{k-1})(x - x_{k+1})\cdots(x - x_n)}{(x_k - x_0)\cdots(x_k - x_{k-1})(x_k - x_{k+1})\cdots(x_k - x_n)}.$$



The Lagrange Polynomial: The General Case

Theorem: n-th Lagrange interpolating polynomial

The Lagrange Polynomial: The General Case

Theorem: *n*-th Lagrange interpolating polynomial

If $x_0, x_1, \ldots, x_n$ are $n+1$ distinct numbers and f is a function whose values are given at these numbers,

The Lagrange Polynomial: The General Case

Theorem: ***n***-th Lagrange interpolating polynomial

If $x_0, x_1, \ldots, x_n$ are $n+1$ distinct numbers and f is a function whose values are given at these numbers, then a unique polynomial $P(x)$ of degree at most n exists with

$$f(x_k) = P(x_k), \quad \text{for each } k = 0, 1, \ldots, n.$$

The Lagrange Polynomial: The General Case

Theorem: *n*-th Lagrange interpolating polynomial

If $x_0, x_1, \ldots, x_n$ are $n+1$ distinct numbers and f is a function whose values are given at these numbers, then a unique polynomial $P(x)$ of degree at most n exists with

$$f(x_k) = P(x_k), \quad \text{for each } k = 0, 1, \ldots, n.$$

This polynomial is given by

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x)$$

where, for each $k = 0, 1, \ldots, n$, $L_{n,k}(x)$ is defined as follows:

The Lagrange Polynomial: The General Case

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x)$$

Definition of $L_{n,k}(x)$

$$\begin{aligned} L_{n,k}(x) &= \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)} \\ &= \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)} \end{aligned}$$

We will write $L_{n,k}(x)$ simply as $L_k(x)$ when there is no confusion as to its degree.

Outline

The Lagrange Polynomial: 2nd Degree Polynomial

Example: $f(x) = \frac{1}{x}$

The Lagrange Polynomial: 2nd Degree Polynomial

Example: $f(x) = \frac{1}{x}$

(a) Use the numbers (called nodes) $x_0 = 2$, $x_1 = 2.75$ and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$.

The Lagrange Polynomial: 2nd Degree Polynomial

Example: $f(x) = \frac{1}{x}$

(a) Use the numbers (called nodes) $x_0 = 2$, $x_1 = 2.75$ and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$.

(b) Use this polynomial to approximate $f(3) = \frac{1}{3}$.

The Lagrange Polynomial: 2nd Degree Polynomial

Part (a): Solution

The Lagrange Polynomial: 2nd Degree Polynomial

Part (a): Solution

We first determine the coefficient polynomials $L_0(x)$, $L_1(x)$, and $L_2(x)$:

$$L_0(x) = \frac{(x-2.75)(x-4)}{(2-2.5)(2-4)} = \frac{2}{3}(x-2.75)(x-4)$$

$$L_1(x) = \frac{(x-2)(x-4)}{(2.75-2)(2.75-4)} = -\frac{16}{15}(x-2)(x-4)$$

$$L_2(x) = \frac{(x-2)(x-2.75)}{(4-2)(4-2.5)} = \frac{2}{5}(x-2)(x-2.75)$$

The Lagrange Polynomial: 2nd Degree Polynomial

Part (a): Solution

We first determine the coefficient polynomials $L_0(x)$, $L_1(x)$, and $L_2(x)$:

$$L_0(x) = \frac{(x-2.75)(x-4)}{(2-2.5)(2-4)} = \frac{2}{3}(x-2.75)(x-4)$$

$$L_1(x) = \frac{(x-2)(x-4)}{(2.75-2)(2.75-4)} = -\frac{16}{15}(x-2)(x-4)$$

$$L_2(x) = \frac{(x-2)(x-2.75)}{(4-2)(4-2.5)} = \frac{2}{5}(x-2)(x-2.75)$$

Also, since $f(x) = \frac{1}{x}$:

$$f(x_0) = f(2) = 1/2, \qquad f(x_1) = f(2.75) = 4/11, \qquad f(x_2) = f(4) = 1/4$$

The Lagrange Polynomial: 2nd Degree Polynomial

Part (a): Solution (Cont'd)

Therefore, we obtain

$$
\begin{aligned}
P(x) &= \sum_{k=0}^{2} f(x_k)L_k(x) \\
&= \frac{1}{3}(x-2.75)(x-4) - \frac{64}{165}(x-2)(x-4) + \frac{1}{10}(x-2)(x-2.75) \\
&= \frac{1}{22}x^2 - \frac{35}{88}x + \frac{49}{44}.
\end{aligned}
$$

The Lagrange Polynomial: 2nd Degree Polynomial

$$P(x) = \frac{1}{22}x^2 - \frac{35}{88}x + \frac{49}{44}$$

(b) Use this polynomial to approximate $f(3) = \frac{1}{3}$.

Part (b): Solution

The Lagrange Polynomial: 2nd Degree Polynomial

$$P(x) = \frac{1}{22}x^2 - \frac{35}{88}x + \frac{49}{44}$$

(b) Use this polynomial to approximate $f(3) = \frac{1}{3}$.

Part (b): Solution

An approximation to $f(3) = \frac{1}{3}$ is

$$f(3) \approx P(3) = \frac{9}{22} - \frac{105}{88} + \frac{49}{44} = \frac{29}{88} \approx 0.32955.$$

The Lagrange Polynomial: 2nd Degree Polynomial

$$P(x) = \frac{1}{22}x^2 - \frac{35}{88}x + \frac{49}{44}$$

(b) Use this polynomial to approximate $f(3) = \frac{1}{3}$.

Part (b): Solution

An approximation to $f(3) = \frac{1}{3}$ is

$$f(3) \approx P(3) = \frac{9}{22} - \frac{105}{88} + \frac{49}{44} = \frac{29}{88} \approx 0.32955.$$

Earlier, we we found that no Taylor polynomial expanded about $x_0 = 1$ could be used to reasonably approximate $f(x) = 1/x$ at $x = 3$.

Second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$