Initial-Value Problems for ODEs

Predictor-Corrector Methods

Numerical Analysis (9th Edition)

R L Burden & J D Faires

Beamer Presentation Slides
prepared by
John Carroll
Dublin City University

Outline

1. The Rationale for Predictor-Corrector Methods

Outline

Outline

Outline

Outline

The Rationale for Predictor-Corrector Methods

The Rationale for Predictor-Corrector Methods

Implicit versus Explicit Methods

- In general, an implicit Adams-Moulton method gives better results than the explicit Adams-Bashforth method of the same order.

The Rationale for Predictor-Corrector Methods

Implicit versus Explicit Methods

- In general, an implicit Adams-Moulton method gives better results than the explicit Adams-Bashforth method of the same order.
- However, implicit methods have the inherent weakness of first having to convert the method algebraically to an explicit representation for w_{i+1}.

The Rationale for Predictor-Corrector Methods

Implicit versus Explicit Methods

- In general, an implicit Adams-Moulton method gives better results than the explicit Adams-Bashforth method of the same order.
- However, implicit methods have the inherent weakness of first having to convert the method algebraically to an explicit representation for w_{i+1}.
- This procedure is not always possible, as can be seen from the following example.

The Rationale for Predictor-Corrector Methods

The Rationale for Predictor-Corrector Methods

Implicit versus Explicit Methods (Cont'd)

- Consider the elementary initial-value problem

$$y' = e^y, \qquad 0 \leq t \leq 0.25, \qquad y(0) = 1$$

The Rationale for Predictor-Corrector Methods

Implicit versus Explicit Methods (Cont'd)

- Consider the elementary initial-value problem

$$y' = e^y, \qquad 0 \leq t \leq 0.25, \qquad y(0) = 1$$

Because $f(t, y) = e^y$, the three-step Adams-Moulton method has

$$w_{i+1} = w_i + \frac{h}{24}\left[9e^{w_{i+1}} + 19e^{w_i} - 5e^{w_{i-1}} + e^{w_{i-2}}\right]$$

as its difference equation, and this equation cannot be algebraically solved for w_{i+1}.

The Rationale for Predictor-Corrector Methods

Implicit versus Explicit Methods (Cont'd)

- Consider the elementary initial-value problem

$$y' = e^y, \qquad 0 \le t \le 0.25, \qquad y(0) = 1$$

  Because $f(t, y) = e^y$, the three-step Adams-Moulton method has

$$w_{i+1} = w_i + \frac{h}{24}\left[9e^{w_{i+1}} + 19e^{w_i} - 5e^{w_{i-1}} + e^{w_{i-2}}\right]$$

  as its difference equation, and this equation cannot be algebraically solved for w_{i+1}.

- One could use Newton's method or the secant method to approximate w_{i+1}, but this complicates the procedure considerably.

Outline

Predictor-Corrector Methods

Combining Explicit & Implicit Methods

Predictor-Corrector Methods

Combining Explicit & Implicit Methods

- In practice, implicit multistep methods are not used as described earlier.

Predictor-Corrector Methods

Combining Explicit & Implicit Methods

- In practice, implicit multistep methods are not used as described earlier.
- Rather, they are used to improve approximations obtained by explicit methods.

Predictor-Corrector Methods

Combining Explicit & Implicit Methods

- In practice, implicit multistep methods are not used as described earlier.
- Rather, they are used to improve approximations obtained by explicit methods.
- The combination of an explicit method to predict and an implicit to improve the prediction is called a predictor-corrector method.

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- Consider the following fourth-order method for solving an initial-value problem.
- The first step is to calculate the starting values w_0, w_1, w_2, and w_3 for the 4-step explicit Adams-Bashforth method.

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- Consider the following fourth-order method for solving an initial-value problem.
- The first step is to calculate the starting values w_0, w_1, w_2, and w_3 for the 4-step explicit Adams-Bashforth method. To do this, we use a 4th-order one-step method, the Runge-Kutta method of order 4.

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- Consider the following fourth-order method for solving an initial-value problem.
- The first step is to calculate the starting values w_0, w_1, w_2, and w_3 for the 4-step explicit Adams-Bashforth method. To do this, we use a 4th-order one-step method, the Runge-Kutta method of order 4.
- The next step is to calculate an approximation, w_4^p, to $y(t_4)$ using the explicit Adams-Bashforth method as predictor:

$$w_4^p = w_3 + \frac{h}{24}[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)]$$

A 4th-order Predictor-Corrector Method

$$w_4^p = w_3 + \frac{h}{24}\left[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)\right]$$

Combining Explicit & Implicit Methods (Cont'd)

A 4th-order Predictor-Corrector Method

$$w_4^p = w_3 + \frac{h}{24}[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)]$$

Combining Explicit & Implicit Methods (Cont'd)

- This approximation is improved by inserting w_4^p in the right side of the three-step implicit Adams-Moulton method and using that method as a corrector. This gives

$$w_4^c = w_3 + \frac{h}{24}\left[9f(t_4, w_4^p) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right]$$

A 4th-order Predictor-Corrector Method

$$w_4^p = w_3 + \frac{h}{24}\left[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)\right]$$

Combining Explicit & Implicit Methods (Cont'd)

- This approximation is improved by inserting w_4^p in the right side of the three-step implicit Adams-Moulton method and using that method as a corrector. This gives

$$w_4^c = w_3 + \frac{h}{24}\left[9f(t_4, w_4^p) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right]$$

- The only new function evaluation required in this procedure is $f(t_4, w_4^p)$ in the corrector equation; all the other values of f have been calculated for earlier approximations.

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- The value w_4^c is then used as the approximation to $y(t_4)$, . . .

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- The value w_4^c is then used as the approximation to $y(t_4)$, ...
- and the technique of using the Adams-Bashforth method as a predictor and the Adams-Moulton method as a corrector is repeated to find w_5^p and w_5^c, the initial and final approximations to $y(t_5)$.

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- The value w_4^c is then used as the approximation to $y(t_4)$, ...
- and the technique of using the Adams-Bashforth method as a predictor and the Adams-Moulton method as a corrector is repeated to find w_5^p and w_5^c, the initial and final approximations to $y(t_5)$.
- This process is continued until we obtain an approximation w_N^c to $y(t_N) = y(b)$.

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- The value w_4^c is then used as the approximation to $y(t_4)$, ...
- and the technique of using the Adams-Bashforth method as a predictor and the Adams-Moulton method as a corrector is repeated to find w_5^p and w_5^c, the initial and final approximations to $y(t_5)$.
- This process is continued until we obtain an approximation w_N^c to $y(t_N) = y(b)$.
- Improved approximations to $y(t_{i+1})$ might be obtained by iterating the Adams-Moulton formula, but these converge to the approximation given by the implicit formula rather than to the solution $y(t_{i+1})$.

A 4th-order Predictor-Corrector Method

Combining Explicit & Implicit Methods (Cont'd)

- The value w_4^c is then used as the approximation to $y(t_4)$, ...
- and the technique of using the Adams-Bashforth method as a predictor and the Adams-Moulton method as a corrector is repeated to find w_5^p and w_5^c, the initial and final approximations to $y(t_5)$.
- This process is continued until we obtain an approximation w_N^c to $y(t_N) = y(b)$.
- Improved approximations to $y(t_{i+1})$ might be obtained by iterating the Adams-Moulton formula, but these converge to the approximation given by the implicit formula rather than to the solution $y(t_{i+1})$.
- Hence it is usually more efficient to use a reduction in the step size if improved accuracy is needed.

Outline

Adams 4th Order Predictor-Corrector Algorithm

Basic Components

Adams 4th Order Predictor-Corrector Algorithm

Basic Components

The following algorithm is based on the 4th-order, 4-step Adams-Bashforth method as predictor

$$w_{i+1}^{p} = w_3 + \frac{h}{24}[55f(t_i, w_i) - 59f(t_{i-1}, w_{i-1}) + 37f(t_{i-2}, w_{i-2}) - 9f(t_{i-3}, w_{i-3})]$$

Adams 4th Order Predictor-Corrector Algorithm

Basic Components

The following algorithm is based on the 4th-order, 4-step Adams-Bashforth method as predictor

$$w_{i+1}^{p} = w_3 + \frac{h}{24}\left[55f(t_i, w_i) - 59f(t_{i-1}, w_{i-1}) + 37f(t_{i-2}, w_{i-2}) - 9f(t_{i-3}, w_{i-3})\right]$$

and one iteration of the 4th-order, 3-step Adams-Moulton method as corrector,

$$w_{i+1}^{c} = w_3 + \frac{h}{24}\left[9f(t_{i+1}, w_{i+1}^{p}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\right]$$

Adams 4th Order Predictor-Corrector Algorithm

Basic Components

The following algorithm is based on the 4th-order, 4-step Adams-Bashforth method as predictor

$$w_{i+1}^{p} = w_3 + \frac{h}{24}\left[55f(t_i, w_i) - 59f(t_{i-1}, w_{i-1}) + 37f(t_{i-2}, w_{i-2}) - 9f(t_{i-3}, w_{i-3})\right]$$

and one iteration of the 4th-order, 3-step Adams-Moulton method as corrector,

$$w_{i+1}^{c} = w_3 + \frac{h}{24}\left[9f(t_{i+1}, w_{i+1}^{p}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\right]$$

with the starting values obtained from the 4th-order Runge-Kutta method.

Adams 4th Order Predictor-Corrector Algorithm

To approximate the solution of the initial-value problem

$$y' = f(t, y), \qquad a \leq t \leq b, \qquad y(a) = \alpha$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$:

Adams 4th Order Predictor-Corrector Algorithm

To approximate the solution of the initial-value problem

$$y' = f(t, y), \qquad a \leq t \leq b, \qquad y(a) = \alpha$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$:

INPUT endpoints a, b; integer N; initial condition α

OUTPUT approximation w to y at the $(N + 1)$ values of t

Step 1 Set $h = (b - a)/N$

$t_0 = a$

$w_0 = \alpha$

OUTPUT (t_0, w_0)

Adams 4th Order Predictor-Corrector Algorithm

To approximate the solution of the initial-value problem

$$y' = f(t, y), \qquad a \le t \le b, \qquad y(a) = \alpha$$

at $(N+1)$ equally spaced numbers in the interval $[a, b]$:

INPUT endpoints a, b; integer N; initial condition α
OUTPUT approximation w to y at the $(N+1)$ values of t

Step 1 Set $h = (b-a)/N$
$t_0 = a$
$w_0 = \alpha$
OUTPUT (t_0, w_0)

Step 2 For $i = 1, 2, 3$, do Steps 3–5
(Compute starting values using Runge-Kutta method)

4th Order Predictor-Corrector Algorithm: Steps 3–6

Step 3 Set $K_1 = hf(t_{i-1}, w_{i-1})$

$$K_2 = hf(t_{i-1} + h/2, w_{i-1} + K_1/2)$$
$$K_3 = hf(t_{i-1} + h/2, w_{i-1} + K_2/2)$$
$$K_4 = hf(t_{i-1} + h, w_{i-1} + K_3)$$

4th Order Predictor-Corrector Algorithm: Steps 3–6

Step 3 Set $K_1 = hf(t_{i-1}, w_{i-1})$

$$K_2 = hf(t_{i-1} + h/2, w_{i-1} + K_1/2)$$

$$K_3 = hf(t_{i-1} + h/2, w_{i-1} + K_2/2)$$

$$K_4 = hf(t_{i-1} + h, w_{i-1} + K_3)$$

Step 4 Set $w_i = w_{i-1} + (K_1 + 2K_2 + 2K_3 + K_4)/6$

$$t_i = a + ih$$

4th Order Predictor-Corrector Algorithm: Steps 3–6

Step 3 Set $K_1 = hf(t_{i-1}, w_{i-1})$
$K_2 = hf(t_{i-1} + h/2, w_{i-1} + K_1/2)$
$K_3 = hf(t_{i-1} + h/2, w_{i-1} + K_2/2)$
$K_4 = hf(t_{i-1} + h, w_{i-1} + K_3)$

Step 4 Set $w_i = w_{i-1} + (K_1 + 2K_2 + 2K_3 + K_4)/6$
$t_i = a + ih$

Step 5 OUTPUT (t_i, w_i)

4th Order Predictor-Corrector Algorithm: Steps 3–6

Step 3 Set $K_1 = hf(t_{i-1}, w_{i-1})$

$K_2 = hf(t_{i-1} + h/2, w_{i-1} + K_1/2)$

$K_3 = hf(t_{i-1} + h/2, w_{i-1} + K_2/2)$

$K_4 = hf(t_{i-1} + h, w_{i-1} + K_3)$

Step 4 Set $w_i = w_{i-1} + (K_1 + 2K_2 + 2K_3 + K_4)/6$

$t_i = a + ih$

Step 5 OUTPUT (t_i, w_i)

Step 6 For $i = 4, \ldots, N$ do Steps 7–10

4th Order Predictor-Corrector Algorithm: Steps 8–11

Step 6 For $i = 4, \ldots, N$ do Steps 7–10

Step 7 Set $t = a + ih$

$$w = w_3 + h[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)]/24 \qquad (Predict \quad w_i)$$

$$w = w_3 + h[9f(t, w) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)]/24 \qquad (Correct \quad w_i)$$

4th Order Predictor-Corrector Algorithm: Steps 8–11

Step 6 For $i = 4, \ldots, N$ do Steps 7–10

Step 7 Set $t = a + ih$

$$w = w_3 + h[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) -9f(t_0, w_0)]/24 \qquad (Predict \quad w_i)$$

$$w = w_3 + h[9f(t, w) + 19f(t_3, w_3) - 5f(t_2, w_2) +f(t_1, w_1)]/24 \qquad (Correct \quad w_i)$$

Step 8 OUTPUT (t, w)

4th Order Predictor-Corrector Algorithm: Steps 8–11

Step 6 For $i = 4, \ldots, N$ do Steps 7–10

Step 7 Set $t = a + ih$

$$w = w_3 + h[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)]/24 \quad (Predict \quad w_i)$$

$$w = w_3 + h[9f(t, w) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)]/24 \quad (Correct \quad w_i)$$

Step 8 OUTPUT (t, w)

Step 9 For $j = 0, 1, 2$

set $t_j = t_{j+1}$ (*Prepare for next iteration*)

$w_j = w_{j+1}$

4th Order Predictor-Corrector Algorithm: Steps 8–11

Step 6 For $i = 4, \ldots, N$ do Steps 7–10

Step 7 Set $t = a + ih$

$$w = w_3 + h[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)]/24 \quad (Predict \quad w_i)$$

$$w = w_3 + h[9f(t, w) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)]/24 \quad (Correct \quad w_i)$$

Step 8 OUTPUT (t, w)

Step 9 For $j = 0, 1, 2$

set $t_j = t_{j+1}$ *(Prepare for next iteration)*

$w_j = w_{j+1}$

Step 10 Set $t_3 = t$

$w_3 = w$

4th Order Predictor-Corrector Algorithm: Steps 8–11

Step 6 For $i = 4, \ldots, N$ do Steps 7–10

Step 7 Set $t = a + ih$

$$w = w_3 + h[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)]/24 \qquad (Predict \quad w_i)$$

$$w = w_3 + h[9f(t, w) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)]/24 \qquad (Correct \quad w_i)$$

Step 8 OUTPUT (t, w)

Step 9 For $j = 0, 1, 2$

set $t_j = t_{j+1}$ *(Prepare for next iteration)*

$w_j = w_{j+1}$

Step 10 Set $t_3 = t$

$w_3 = w$

Step 11 STOP

Outline

Applying the 4th Order Predictor-Corrector Method

Example

Apply the Adams fourth-order predictor-corrector method with $h = 0.2$ and starting values from the Runge-Kutta fourth order method to the initial-value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5$$

Applying the 4th Order Predictor-Corrector Method

Example

Apply the Adams fourth-order predictor-corrector method with $h = 0.2$ and starting values from the Runge-Kutta fourth order method to the initial-value problem

$$y' = y - t^2 + 1, \qquad 0 \leq t \leq 2, \qquad y(0) = 0.5$$

This is a continuation and modification of an earlier problem where we found that the starting approximations from Runge-Kutta are

$$\begin{aligned} y(0) = w_0 &= 0.5 \\ y(0.2) \approx w_1 &= 0.8292933 \\ y(0.4) \approx w_2 &= 1.2140762 \\ y(0.6) \approx w_3 &= 1.6489220 \end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (1/6)

Applying the 4th Order Predictor-Corrector Method

Solution (1/6)

For the 4th-order Adams-Bashforth, we obtained

$$\begin{aligned} y(0.8) \approx w_4^p &= w_3 + \frac{0.2}{24}(55f(0.6, w_3) - 59f(0.4, w_2) + 37f(0.2, w_1) \\ &\quad -9f(0, w_0)) \end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (1/6)

For the 4th-order Adams-Bashforth, we obtained

$$
\begin{aligned}
y(0.8) \approx w_4^p &= w_3 + \frac{0.2}{24}(55f(0.6, w_3) - 59f(0.4, w_2) + 37f(0.2, w_1) \\
&\qquad -9f(0, w_0)) \\
&= 1.6489220 + \frac{0.2}{24}(55f(0.6, 1.6489220) \\
&\qquad -59f(0.4, 1.2140762) + 37f(0.2, 0.8292933) \\
&\qquad -9f(0, 0.5))
\end{aligned}
$$

Applying the 4th Order Predictor-Corrector Method

Solution (1/6)

For the 4th-order Adams-Bashforth, we obtained

$$
\begin{aligned}
y(0.8) \approx w_4^p &= w_3 + \frac{0.2}{24}(55f(0.6, w_3) - 59f(0.4, w_2) + 37f(0.2, w_1) \\
&\qquad -9f(0, w_0)) \\
&= 1.6489220 + \frac{0.2}{24}(55f(0.6, 1.6489220) \\
&\qquad -59f(0.4, 1.2140762) + 37f(0.2, 0.8292933) \\
&\qquad -9f(0, 0.5)) \\
&= 1.6489220 + 0.0083333(55(2.2889220) \\
&\qquad -59(2.0540762) + 37(1.7892933) - 9(1.5))
\end{aligned}
$$

Applying the 4th Order Predictor-Corrector Method

Solution (1/6)

For the 4th-order Adams-Bashforth, we obtained

$$\begin{aligned}
y(0.8) \approx w_4^p &= w_3 + \frac{0.2}{24}(55f(0.6, w_3) - 59f(0.4, w_2) + 37f(0.2, w_1) \\
&\qquad -9f(0, w_0)) \\
&= 1.6489220 + \frac{0.2}{24}(55f(0.6, 1.6489220) \\
&\qquad -59f(0.4, 1.2140762) + 37f(0.2, 0.8292933) \\
&\qquad -9f(0, 0.5)) \\
&= 1.6489220 + 0.0083333(55(2.2889220) \\
&\qquad -59(2.0540762) + 37(1.7892933) - 9(1.5)) \\
&= 2.1272892
\end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (2/6)

Applying the 4th Order Predictor-Corrector Method

Solution (2/6)

We will now use w_4^p as the predictor of the approximation to $y(0.8)$ and determine the corrected value $w_4^c = w_c$, from the implicit Adams-Moulton method:

$$y(0.8) \approx w_4^c = w_3 + \frac{0.2}{24}\left(9f(0.8, w_4^p) + 19f(0.6, w_3) - 5f(0.4, w_2) + f(0.2, w_1)\right)$$

Applying the 4th Order Predictor-Corrector Method

Solution (2/6)

We will now use w_4^p as the predictor of the approximation to $y(0.8)$ and determine the corrected value $w_4^c = w_c$, from the implicit Adams-Moulton method:

$$\begin{aligned} y(0.8) \approx w_4^c &= w_3 + \frac{0.2}{24}\left(9f(0.8, w_4^p) + 19f(0.6, w_3) - 5f(0.4, w_2)\right. \\ &\quad \left. + f(0.2, w_1)\right) \\ &= 1.6489220 + \frac{0.2}{24}(9f(0.8, 2.1272892) + 19f(0.6, 1.6489220) \\ &\quad -5f(0.4, 1.2140762) + f(0.2, 0.8292933)) \end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (2/6)

We will now use w_4^p as the predictor of the approximation to $y(0.8)$ and determine the corrected value $w_4^c = w_c$, from the implicit Adams-Moulton method:

$$\begin{aligned} y(0.8) &\approx w_4^c = w_3 + \frac{0.2}{24}\left(9f(0.8, w_4^p) + 19f(0.6, w_3) - 5f(0.4, w_2)\right. \\ &\qquad \left. + f(0.2, w_1)\right) \\ &= 1.6489220 + \frac{0.2}{24}(9f(0.8, 2.1272892) + 19f(0.6, 1.6489220) \\ &\qquad - 5f(0.4, 1.2140762) + f(0.2, 0.8292933)) \\ &= 1.6489220 + 0.0083333\,(9(2.4872892) \\ &\qquad + 19(2.2889220) - 5(2.0540762) + (1.7892933)) \end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (2/6)

We will now use w_4^p as the predictor of the approximation to $y(0.8)$ and determine the corrected value $w_4^c = w_c$, from the implicit Adams-Moulton method:

$$\begin{aligned}
y(0.8) &\approx w_4^c = w_3 + \frac{0.2}{24}\left(9f(0.8, w_4^p) + 19f(0.6, w_3) - 5f(0.4, w_2)\right. \\
&\qquad \left. + f(0.2, w_1)\right) \\
&= 1.6489220 + \frac{0.2}{24}(9f(0.8, 2.1272892) + 19f(0.6, 1.6489220) \\
&\qquad - 5f(0.4, 1.2140762) + f(0.2, 0.8292933)) \\
&= 1.6489220 + 0.0083333\,(9(2.4872892) \\
&\qquad + 19(2.2889220) - 5(2.0540762) + (1.7892933)) \\
&= 2.1272056
\end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (3/6)

Applying the 4th Order Predictor-Corrector Method

Solution (3/6)

Now we use this approximation (noting that $w_4 = w_4^c$) to determine the predictor, w_5^p, for $y(1.0)$:

$$y(1.0) \approx w_5^p = w_4 + \frac{0.2}{24}(55f(0.8, w_4) - 59f(0.6, w_3) + 37f(0.4, w_2) - 9f(0.2, w_1))$$

Applying the 4th Order Predictor-Corrector Method

Solution (3/6)

Now we use this approximation (noting that $w_4 = w_4^c$) to determine the predictor, w_5^p, for $y(1.0)$:

$$\begin{aligned} y(1.0) &\approx w_5^p = w_4 + \frac{0.2}{24}(55f(0.8, w_4) - 59f(0.6, w_3) + 37f(0.4, w_2) \\ &\qquad -9f(0.2, w_1)) \\ &= 2.1272056 + \frac{0.2}{24}(55f(0.8, 2.1272056) \\ &\qquad -59f(0.6, 1.6489220) + 37f(0.4, 1.2140762) \\ &\qquad -9f(0.2, 0.8292933)) \end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (3/6)

Now we use this approximation (noting that $w_4 = w_4^c$) to determine the predictor, w_5^p, for $y(1.0)$:

$$\begin{aligned}
y(1.0) \;\approx\; & w_5^p = w_4 + \frac{0.2}{24}\left(55f(0.8, w_4) - 59f(0.6, w_3) + 37f(0.4, w_2)\right. \\
& \qquad \left. -9f(0.2, w_1)\right) \\
= \; & 2.1272056 + \frac{0.2}{24}(55f(0.8, 2.1272056) \\
& \qquad -59f(0.6, 1.6489220) + 37f(0.4, 1.2140762) \\
& \qquad -9f(0.2, 0.8292933)) \\
= \; & 2.1272056 + 0.0083333\,(55(2.4872056) - 59(2.2889220) \\
& \qquad +37(2.0540762) - 9(1.7892933))
\end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (3/6)

Now we use this approximation (noting that $w_4 = w_4^c$) to determine the predictor, w_5^p, for $y(1.0)$:

$$\begin{aligned}
y(1.0) &\approx w_5^p = w_4 + \frac{0.2}{24}\left(55f(0.8, w_4) - 59f(0.6, w_3) + 37f(0.4, w_2)\right. \\
&\qquad \left. - 9f(0.2, w_1)\right) \\
&= 2.1272056 + \frac{0.2}{24}\left(55f(0.8, 2.1272056)\right. \\
&\qquad - 59f(0.6, 1.6489220) + 37f(0.4, 1.2140762) \\
&\qquad \left. - 9f(0.2, 0.8292933)\right) \\
&= 2.1272056 + 0.0083333\left(55(2.4872056) - 59(2.2889220)\right. \\
&\qquad \left. + 37(2.0540762) - 9(1.7892933)\right) \\
&= 2.6409314
\end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (4/6)

and correct this with

$$y(1.0) \approx w_5^c = w_4 + \frac{0.2}{24}\left(9f(1.0, w_5^p) + 19f(0.8, w_4) - 5f(0.6, w_3) + f(0.4, w_2)\right)$$

Applying the 4th Order Predictor-Corrector Method

Solution (4/6)

and correct this with

$$\begin{aligned} y(1.0) &\approx w_5^c = w_4 + \frac{0.2}{24}\left(9f(1.0, w_5^p) + 19f(0.8, w_4) - 5f(0.6, w_3)\right. \\ &\quad \left. + f(0.4, w_2)\right) \\ &= 2.1272056 + \frac{0.2}{24}(9f(1.0, 2.6409314) \\ &\quad + 19f(0.8, 2.1272892) - 5f(0.6, 1.6489220) \\ &\quad + f(0.4, 1.2140762)) \end{aligned}$$

Applying the 4th Order Predictor-Corrector Method

Solution (4/6)

and correct this with

$$
\begin{aligned}
y(1.0) \approx w_5^c &= w_4 + \frac{0.2}{24}\left(9f(1.0, w_5^p) + 19f(0.8, w_4) - 5f(0.6, w_3)\right. \\
&\quad \left. + f(0.4, w_2)\right) \\
&= 2.1272056 + \frac{0.2}{24}(9f(1.0, 2.6409314) \\
&\quad + 19f(0.8, 2.1272892) - 5f(0.6, 1.6489220) \\
&\quad + f(0.4, 1.2140762)) \\
&= 2.1272056 + 0.0083333\,(9(2.6409314) + 19(2.4872056) \\
&\quad - 5(2.2889220) + (2.0540762))
\end{aligned}
$$

Applying the 4th Order Predictor-Corrector Method

Solution (4/6)

and correct this with

$$
\begin{aligned}
y(1.0) \approx w_5^c &= w_4 + \frac{0.2}{24}\left(9f(1.0, w_5^p) + 19f(0.8, w_4) - 5f(0.6, w_3)\right. \\
&\quad \left. + f(0.4, w_2)\right) \\
&= 2.1272056 + \frac{0.2}{24}(9f(1.0, 2.6409314) \\
&\quad + 19f(0.8, 2.1272892) - 5f(0.6, 1.6489220) \\
&\quad + f(0.4, 1.2140762)) \\
&= 2.1272056 + 0.0083333\,(9(2.6409314) + 19(2.4872056) \\
&\quad - 5(2.2889220) + (2.0540762)) \\
&= 2.6408286
\end{aligned}
$$

Applying the 4th Order Predictor-Corrector Method

Solution (5/6): Accuracy Check

Earlier, we found that using the explicit Adams-Bashforth method alone produced results that were inferior to those of Runge-Kutta.

Applying the 4th Order Predictor-Corrector Method

Solution (5/6): Accuracy Check

Earlier, we found that using the explicit Adams-Bashforth method alone produced results that were inferior to those of Runge-Kutta. However, these approximations to $y(0.8)$ and $y(1.0)$ are accurate to within:

$$\begin{aligned} |2.1272295 - 2.1272056| &= 2.39 \times 10^{-5} \quad \text{and} \\ |2.6408286 - 2.6408591| &= 3.05 \times 10^{-5} \end{aligned}$$

respectively,

Applying the 4th Order Predictor-Corrector Method

Solution (5/6): Accuracy Check

Earlier, we found that using the explicit Adams-Bashforth method alone produced results that were inferior to those of Runge-Kutta. However, these approximations to $y(0.8)$ and $y(1.0)$ are accurate to within:

$$\begin{aligned} |2.1272295 - 2.1272056| &= 2.39 \times 10^{-5} \quad \text{and} \\ |2.6408286 - 2.6408591| &= 3.05 \times 10^{-5} \end{aligned}$$

respectively, compared to those of Runge-Kutta, which were accurate, respectively, to within:

$$\begin{aligned} |2.1272027 - 2.1272892| &= 2.69 \times 10^{-5} \quad \text{and} \\ |2.6408227 - 2.6408591| &= 3.64 \times 10^{-5} \end{aligned}$$

4th Order Predictor-Corrector Method (6/6)

The remaining approximations were generated using the Adams 4th-Order Predictor-Corrector Algorithm:

t_i	$y_i = y(t_i)$	w_i	$\|y_i - w_i\|$
0.0	0.5000000	0.5000000	0
0.2	0.8292986	0.8292933	0.0000053
0.4	1.2140877	1.2140762	0.0000114
0.6	1.6489406	1.6489220	0.0000186
0.8	2.1272295	2.1272056	0.0000239
1.0	2.6408591	2.6408286	0.0000305
1.2	3.1799415	3.1799026	0.0000389
1.4	3.7324000	3.7323505	0.0000495
1.6	4.2834838	4.2834208	0.0000630
1.8	4.8151763	4.8150964	0.0000799
2.0	5.3054720	5.3053707	0.0001013

Questions?