Approximating Eigenvalues

Single Value Decomposition

Numerical Analysis (10th Edition)

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Outline

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Rank and Nullity 1

Definition

Let A be an $m \times n$ matrix

- The **Rank** of A, denoted Rank (A) is the number of linearly independent rows in A.
- The **Nullity** of A, denoted Nullity of A, is $n - \mathrm{Nullity}(A)$, and describes the largest set of linearly independent vectors *textbfv* in $\mathbb{R}^n$ for which $A\mathbf{v} = \mathbf{0}$.
- NOTE: The Rank and Nullity of a matrix are important in characterizing the behavior of the matrix.

Rank and Nullity 2

Theorem 9.25

The number of linearly independent rows of an $m \times n$ matrix A is the same as the number of linearly independent columns of A.

Theorem 9.26

Let A be $m \times n$ matrix.

- **(i)** The matrices A^tA and AA^t are symmetric.
- **(ii)** $A = A^tA$.
- **(iii)** $A = A^tA$.
- **(iv)** The eigenvalues of A^tA are real and nonnegative.
- **(v)** The nonzero eigenvalues of AA^t are the same as the nonzero eigenvalues of A^tA .

Outline

Overview

Our objective is to determine a factorization of the $m \times n$ matrix A, where $m \geq n$, in the form

$$A = U\,S\,V^t,$$

where U is an $m \times m$ orthogonal matrix, V is $n \times n$ an orthogonal matrix, and S is an $m \times n$ diagonal matrix, that is, its only nonzero entries are $(S)_{ii} \equiv s_i \geq 0$, for $i = 1, \ldots, n$.

Construction of the Factorization

1. Construct S in the factorization $A = U\,S\,V^t$
   1. Find the eigenvalues of the $n \times n$ symmetric matrix $A^t A$.
   2. Order them from largest to smallest and denote as $s_1^2 \geq s_2^2 \geq \cdots \geq s_k^2 > s_{k+1} = \cdots = s_n = 0$.
   3. The diagonal entries of D are the square roots of singular values of $s_1, s_2, s_3, \cdots, s_n$.
2. Construct V in the factorization $A = U\,S\,V^t$
   1. Find the associated eigenvectors $v_1, v_2, \cdots, v_n$ for the eigenvectors $s_1, s_2, s_3, \cdots, s_n$.
   2. Normalize the eigenvectors $v_1, v_2, \cdots, v_n$ to obtain the columns of V.
3. Construct U in the factorization $A = U\,S\,V^t$
   1. Compute the first k columns of U: $u_i = \frac{1}{s_i} A v_i$ for $i = 1, 2, \ldots, k$.
   2. Use the Gram-Schmidt process to obtain additional columns .

Alternate Step 3 of USV^t

- Compute the m eigenvalues of AA^t.
- Find the set of m corresponding eigenvectors.
- Normalize these eigenvectors and make them the columns of U.