Math 311

Numerical Methods

2.0: Overview of Chapter

Solutions of Equations in One Variable

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1 Introduction

- Sometimes the solution to an equation cannot be solved algebraically.
- This chapter will talk about several numerical methods to solve equations.
- This includes:
  - finding zeros (or roots)
  - finding intersections of two curves,
  - optimizing a function (maximize or minimize)
- In these cases, we can find an approximation to the exact solutions (to MANY decimal places)

Objective Function

We want to find where

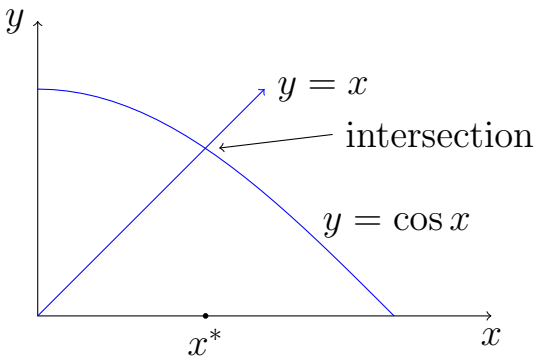
$$f(x) = 0$$

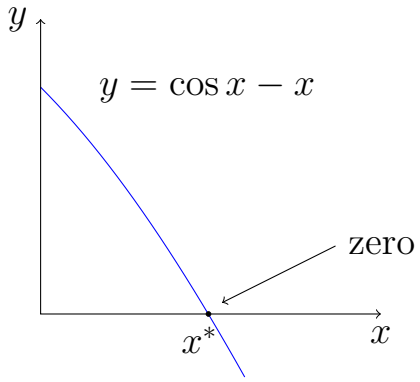
1.1 Example: Finding an intersection

- Suppose we want to find the intersection of the graphs of $y = \cos x$ and $y = x$.
- That is equivalent to finding where $\boldsymbol{f(x) = 0}$, where

$$\boldsymbol{f(x) = \cos x - x.}$$

- Both pictures show the same solution x^*:





2 Fixed Point Solutions

- An equivalent way of solving these problems is through a fixed point function.
- A function $g(x)$ has a fixed point at p if $g(p) = p$.
- Convert the problem from $f(x) = 0$ to $g(x) = x$.
- To find the equivalent $g(x)$ for any $f(x)$, start with $f(x) = 0$ and solve for x in algebraic or sneaky methods.
- For example,
  - $f(x) = \cos x - x = 0$ is equivalent to $g(x) = \cos x = x$ (or $g(x) = \cos^{-1}(x)$)
  - $f(x) = x^2 - 2x + 3 = 0$ is equivalent to $g(x) = \frac{x^2+3}{2} = x$ (or others).
- The selection is an art!

3 Bisection Method

Now, let's start with Bisection method!!