Math 311

Numerical Methods

3.0: Overview of Chapter

Interpolation and Polynomial Approximation

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1 Introduction

- We want to estimate and interpolate functions.
- Taylor Polynomials does NOT DO a good job of this (only fits around ONE point).
- This chapter will talk about several numerical methods to interpolate functions with polynomials. The methods include:
  - Vandermonde Matrices
  - Lagrange Polynomials
  - Neville's Method
  - Divided Differences (several kinds)
  - Hermite Polynomials
  - Cubic Spline Polynomials
  - Parametric Curves (Bézier Curve)

Polynomials are generally: (for finite n)

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

Weierstrass Approximation Theorem

Theorem. *If f is defined and continous on $[a, b]$ and $\varepsilon > 0$ is given, then there exists a polynomial P, defined on $[a, b]$, with the property that*

$$|f(x) - P(x)| < \varepsilon \text{ for all } x \in [a, b]$$