Math 311

Numerical Methods

3.3: Hermite Interpolation

Matching First Derivatives

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Burden and Faires, any ed.

Winter 2024

1 Introduction

- Osculating polynomials are a generalization of both Taylor Polynomials and Lagrange Polynomials.
- What are Osculating polynomials?

Definition. *Let $x_0, x_1, \cdots, x_n$ be $n+1$ distinct numbers in $[a, b]$ and m_i be a non-negative integer associated with x_i for $i = 0, 1, \cdots, n$. Let*

$$m = \max_{0 \leqslant i \leqslant n} m_i \text{ and } f \in C^m[a, b]$$

The osculating polynomial approximating f is the polynomial P of least degree such that

$$\frac{d^k P(x_i)}{dx^k} = \frac{d^k f(x_i)}{dx^k},$$

for each $i = 0, 1, \cdots, n$ and $k = 0, 1, \cdots, m_i$

- Note that when $n = 0$, the osculating polynomial approximating f is simply the m_0^{th} Taylor polynomial for f at x_0
- When $m_i = 0$ for $i = 0, 1, \cdots, n$, the osculating polynomial is the n^{th} Lagrange

polynomials interpolating f on $x_0, x_1, \cdots, x_n$.

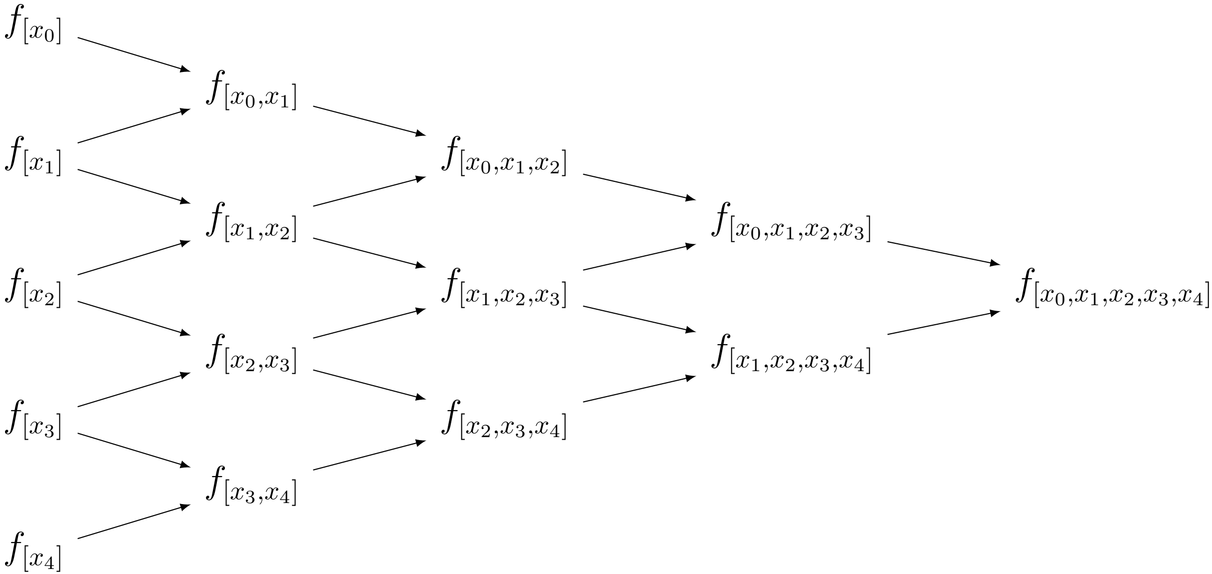
- The case when $m_i = 1$ for each $i = 0, 1, \cdots, n$ gives a class called the Hermite polynomials.
- For a given f, they agree with f at the points $x_0, x_1, \cdots, x_n$ AND they agree with f' at those points as well.
- This gives a much better shape to the approximating polynomial.

n^{th} Hermite Polynomial

Theorem.
$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0)(x - x_1) \cdots (x - x_n)$,
where the constants a_i are solved for.

To obtain the divided-difference coeficients of the interpolatory polynomial $P(x)$ on the $(n + 1)$ distinct numbers, x_0, $x_1, \cdots, x_n$ for the function $f(x)$:

- Input: numbers $x_0, x_1, \cdots, x_n$, plus



– Note that the polynomial coefficients follow the top numbers in the table. All the other numbers are only there to create all the numbers at the top.

X	y	First DD	Second DD	Third DD	Fourth DD
x_0	$f[x_0]$				
		$f[x_0, x_1]$			
x_1	$f[x_1]$		$f[x_0, x_1, x_2]$		
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3]$	
x_2	$f[x_2]$		$f[x_1, x_2, x_3]$		$f[x_0, x_1, x_2, x_3, x_4]$
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4]$	
x_3	$f[x_3]$		$f[x_2, x_3, x_4]$		
		$f[x_3, x_4]$			
x_4	$f[x_4]$				