BAIN / ENGELHARDT

[1.1] Introduction

Mathematical Models

$$V = gt, \quad \text{where } g = 9.8 \ m/s^2$$

(velocity (m/sec) of a body falling from rest in a vacuum.)

This is a deterministic model

Carrying out repeated experiments under ideal conditions would result with the same velocity each time, and this would be predicted by the model.

Probability models

Other phenomena occur by chance, and deterministic won't work.

- number of particles emitted by a radioactive source
- time until failure of a television
- outcome of a game of chance

This is called a stochastic model or probability models

Stochos – Greek word meaning "guess"

[1.2] Notation and Terminology

Experiment - process of obtaining an observed result of some phenomenon. (Like flip a coin)

A performance of an experiment is a trial. (Actually doing it)

An observed outcome of a trial is called an outcome.
(Now recording it)

The set of all possible outcomes is called the sample space
(All outcomes are enumerated)

(Note: one and only one of the possible outcomes will occur on any given trial of the experiment.)

Ex: Flip a coin twice

sample space is $S = \{HH, HT, TH, TT\}$

The experiment is flipping a coin twice

Ex: Change the experiment to flipping a coin twice and recording the number of Heads showing.

The sample space is $S = \{0, 1, 2\}$

Ex: New experiment: Repeatedly toss a coin until a head shows.

The sample space is $S = \{H, TH, TTH, TTTH, TTTTH$

If we count the number of tails until a head shows up.

then $S = \{0, 1, 2, \cdots\}$

EX: A light bulb is tested to see how long it lasts

The sample space is $S = \{t \mid 0 \le t < \infty\}$

If the actual failure time could be measured only to the nearest hour, then the sample space would be

$$S^* = \{0, 1, 2, \ldots\}$$

Note that although S^* may be the observable sample space, we might prefer S to describe the behavior of the light bulb

In other words, the sample space should be described with S even when the observable sample space is S^*

A sample space is finite if it consists of a finite number of outcomes

$$S = \{e_1, e_2, \cdots, e_N\}, \quad N < \infty$$

and it is said to be countably infinite if its outcomes can be placed in a one-to-one correspondence with the positive integers

$$S = \{e_1, e_2, \cdots\}$$

DEF: If a sample space is either finite or countably infinite, then it is called a discrete sample space.

A set that is finite or countably infinite is said to be countable.

An event is a subset of the sample space.

If A is an event, then A has occurred if it contains the outcome that occurred.

For example, suppose $A = \{HH, TT\}$. Then A has occurred if either HH or TT was observed.

A sample space that involves uncountably many outcomes is called a continuous sample space.

Set Notation

Unions → "or" ⇒ Use $\cup$

Intersections → "and" ⇒ Use $\cap$

Ex: A, B, sets

$A \cap B$ ⇒ An event that is in A <u>and</u> in B.

$A \cup B$ ⇒ An event that is in A <u>or</u> in B.

A' (or $\bar{A}$) ⇒ Complement of A. The stuff that does <u>not</u> belong to A.

In A but not in B ⇒ $A \cap B'$

Exactly one of A or B ⇒ $(A \cap B') \cup (A' \cap B)$

De Morgan's Laws

$$A' \cap B' = (A \cup B)'$$

$$A' \cup B' = (A \cap B)'$$

Empty set $\rightarrow \phi$

An event is an elementary event if it contains exactly one outcome of the experiment

Two events A and B are mutually exclusive if the have no events in common and is notated $A \cap B = \phi$

Events $A_1, A_2, \ldots$ are said to be mutually exlcusive if the are pairwise mutually exclusive. That is, if

$(A_i \cap A_j = \phi$, whenever $i \neq j)$

Relative Frequency

If $m(A)$ represents the number of times the event A occurs in M trials, then

$$\boxed{f_A = \frac{m(A)}{M}}$$

represents the relative frequency of the occurrence of A on these trials

If, for an event A, the limit of f_A as M approaches ∞ exists, then one could assign probability to A by

$$P(A) = \lim_{M \to \infty} f_A$$

This expresses a property known as statistical regularity.

When will this limit exist?

Motivating the Axioms of Probabilities

If S is a sample space and $A, B, A_1, A_2, \dots, A_K, \dots$ are all pairwise mutually exclusive events in S, then:

Clearly, $m(A) \geq 0$, $m(S) = M$ $\Rightarrow$ $f_A \geq 0$

$$m(A \cup B) = m(A) + m(B)$$

$$m(A_1 \cup A_2 \cup \cdots \cup A_K \cup \cdots) = m(A_1) + m(A_2) + \cdots + m(A_K) + \cdots$$

So:

1) $f_A \geq 0$

2) $f_S = 1$

3) $f_{A_1 \cup A_2 \cdots \cup A_K \cup \cdots} = f_{A_1} + f_{A_2} + \cdots + f_{A_K} + \cdots$