[1.3] Definition of Probability

$P(A)$ is a set function (that is, it is a function whose domain is a collection of sets, and the range is a subset of the real numbers.

$$P(A) : \underset{\substack{\uparrow \\ \text{collection of sets.}}}{B} \to [0, 1]$$

– Not all set functions are suitable for assigning prob.

<u>DEF:</u> For a given experiment,

S denotes the sample space

$A, A_1, A_2, \ldots$ represent possible events.

A set function $P(A)$ is called the prob of A if the following are satisfied:

$$0 \le P(A) \quad \text{for every } A$$

$$P(S) = 1$$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i) \quad \text{if } A_i \cap A_j = \phi \text{ whenever } i \neq j$$

First consequence:

$P(\phi) = 0$

$S \cup \phi = S$

$P(S \cup \phi) = P(S)$

$P(S) + P(\phi) = P(S)$

$\boxed{P(\phi) = 0}$ ✓

Probability in Discrete Spaces

The assignment of prob in the case of discrete spaces can be reduced to assigning probability to elementary events.

let $\{e_i\}$ be an elementary event.

$$P(\{e_i\}) = p_i$$

So, to satisfy the axioms of prob (def of Prob)

$p_i \geq 0$ for all i.

$$\sum_{\text{all } i} p_i = 1$$

Ex: Coin toss

$S = \{HH, HT, TH, TT\}$

$\Rightarrow$ prob of each event is $1/4$.

$P(HH) = 1/4 = P(HT) = P(TH) = P(TT)$

X = # of heads in 2 toss

$S = \{0, 1, 2\}$ No! $\Rightarrow P(\text{each}) = 1/3$??

$P(X=1) = P(HT) + P(TH)$

$= \frac{1}{4} + \frac{1}{4}$

$= 1/2$

$P(0) = P(2) = 1/4$.

Classical Prob

- finite number of outcomes.
- equally likely outcomes

$S = \{e_1, e_2, \cdots, e_N\}$

$p_1 = p_2 = \cdots = p_N = \frac{1}{N}$

any event

$$P(A) = \frac{n(A)}{N} = \frac{\text{\# of ways A can occur}}{\text{total size of sample space.}}$$

Random Selection

If an object is chosen from a finite collection of distinct objects in such a manner that each object has same prob of being chosen, then we say that the object was chosen at random.