[3.4] Location and Scale Parameters

DEF 3.4.3

Location Scale Parameters

η and $\theta > 0$ are called location-scale parameters if the CDF has the form

$$F(x;\theta,\eta) = F_0\left(\frac{x-\eta}{\theta}\right)$$

OR:

$$f(x;\theta,\eta) = \frac{1}{\theta} f_0\left(\frac{x-\eta}{\theta}\right)$$

$F_0(z)$ represents a completely specified CDF (it does not depend on an unknown parameter)

$f_0(z)$ is the pdf corresponding to F_0

EX: Cauchy dist.

$$f_0(z) = \frac{1}{\pi}\frac{1}{1+z^2}$$

$$f(x;\theta,\eta) = \frac{1}{\theta} f\left(\frac{x-\eta}{\theta}\right) = \frac{1}{\pi\theta}\frac{1}{1+\left(\frac{x-\eta}{\theta}\right)^2}$$

$X \sim CAU(\theta,\eta)$

Note: $E(X)$ d.n.e.
$V(X)$ d.n.e

So η & θ do not represent the mean & std dev.

DEF: Location Parameters

$\eta \to$ location parameter for a dist X if

$$F(x,\eta) = F_0(x-\eta)$$

$$f(x,\eta) = f_0(x-\eta)$$

EX: $f(x,\eta) = e^{-(x-\eta)} \quad x > \eta$

$$x > \eta \Rightarrow \eta < x < \infty \Rightarrow I_{(\eta,\infty)}(x)$$

$$0 < x-\eta < \infty \Rightarrow I_{(0,\infty)}(x-\eta)$$

Thus, $f(x,\eta) = e^{-(x-\eta)} I_{(0,\infty)}(x-\eta)$

$$f_0(x) = e^{x} I_{(0,\infty)}(x) \Rightarrow f_0(x-\eta) = e^{(x-\eta)} I_{(0,\infty)}(x-\eta)$$

It is common for the location parameter to be a measure of central tendency such as the median or mean (doesn't always happen)

DEF: Scale Parameters

$\theta > 0 \Rightarrow$ Scale parameter for a R.V X if

$$F(x;\theta) = F_0\left(\frac{x}{\theta}\right)$$

in other words $f(x,\theta) = f_0\left(\frac{x}{\theta}\right)\left(\frac{1}{\theta}\right) = \frac{1}{\theta} f_0\left(\frac{x}{\theta}\right)$

EX. $X \sim EXP(\theta) \Rightarrow \theta$ is a Scale parameter

$X \sim N(\mu,\sigma^2) \Rightarrow \sigma$ is a Scale parameter

Often σ is a scale parameter. Not Always!

$X \sim WEI(\theta, 2) \Rightarrow \theta$ is a scale parameter, but it is not the std deviation