

[9.4] Large-sample Properties

DEF: Simple consistency.

Let $\{T_n\}$ be a sequence of est. of $\tau(\theta)$. These est. are said to be consistent est. of $\tau(\theta)$ if, for every $\epsilon > 0$,

$$\frac{\sum (x_i - \bar{x})^2}{n} \quad \lim_{n \rightarrow \infty} P[|T_n - \tau(\theta)| < \epsilon] = 1$$

for all $\theta \in \Omega$

DEF: MSE Consistency.

$$\lim_{n \rightarrow \infty} E[T_n - \tau(\theta)]^2 = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} \text{MSE}(T_n) = 0$$

DEF: Asymptotically Unbiased.

T_n est. $\tau(\theta)$

$$\lim_{n \rightarrow \infty} E(T_n) = \tau(\theta)$$

Ex: $\hat{\theta}_n = \frac{n-1}{n} S^2$ $\hat{\theta}_n$ unbiased? No!

$$E(\hat{\theta}_n) = \frac{n-1}{n} \sigma^2$$

Asymptotic Properties of MLEs

Under certain regularity conditions, the MLE $\hat{\theta}_n$ has the following prop.

1. $\hat{\theta}_n$ exists and is unique
2. $\hat{\theta}_n$ is a consistent est of θ .
3. $\hat{\theta}_n$ is asymptotically normal w/ asymptotic mean θ and variance $\frac{1}{n E\left(\frac{\partial}{\partial \theta} \ln f\right)^2} \Rightarrow \hat{\theta}_n \sim N(\theta, \text{CRLB})$
4. $\hat{\theta}$ is asymptotically efficient.

Thm: A sequence $\{T_n\}$ of est of $\tau(\theta)$
is mean squared consis. iff.

$$MSE(T_n) = \underbrace{\text{Var}(T_n)}_{\rightarrow 0} + \underbrace{b(T_n)^2}_{\rightarrow 0}$$

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it is asymptotically unbiased and $\lim_{n \rightarrow \infty} \text{Var}(T_n) = 0$.

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$$\lim_{n \rightarrow \infty} b(T_n) = E(T_n) - \tau(\theta) = 0$$

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Thm: If $\{T_n\}$ is simply consis for $\tau(\theta)$ and if
 $g(t)$ is continuous at each value of $\tau(\theta)$, then

$g(T_n)$ is simply consis. for $g(\tau(\theta))$

Ex: $\hat{\theta}_n = \left(\frac{n-1}{n}\right)s^2 \Rightarrow$ simply consistent?

$\Rightarrow \text{in } \hat{\theta}_n$ simply consistent for $\text{Var}(s^2)$